Effective field theory for the nuclear shell model

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Limits of existence of light Nuclei, ECT*, Oct 25-29 2010

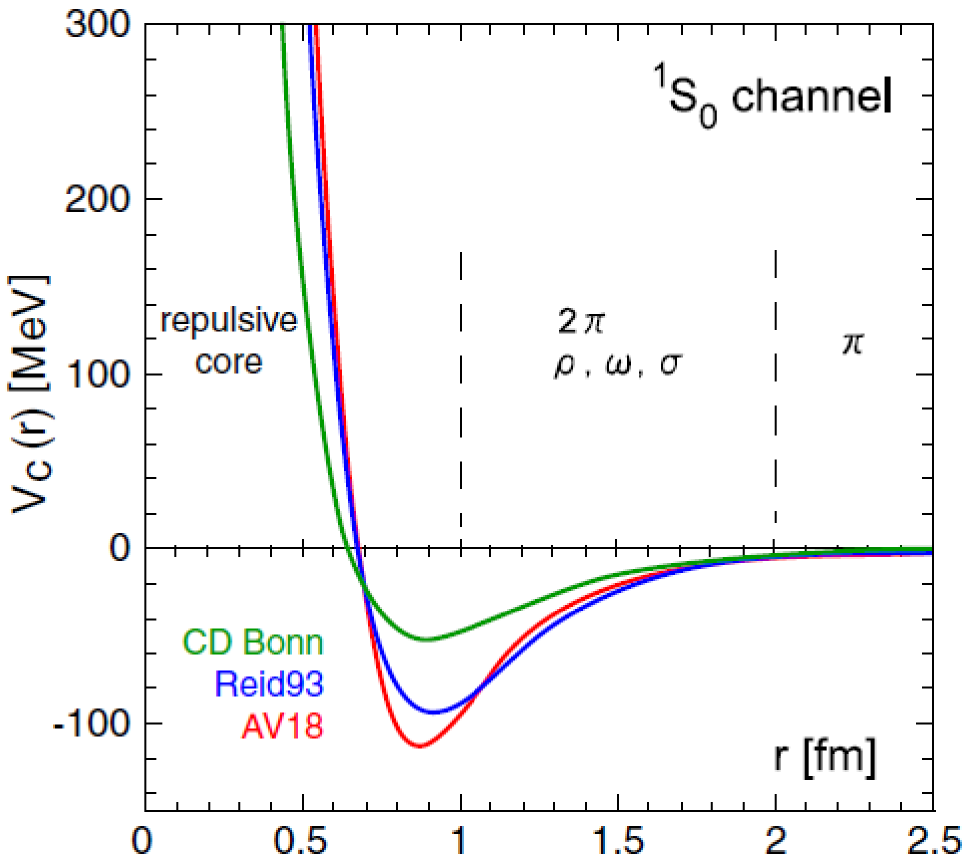


FIG. 1 (color online). Three examples of the modern NN potential in the $^1S_0$ (spin singlet and s-wave) channel: CD-Bonn [17], Reid93 [18], and AV18 [19] from the top at $r = 0.8$ fm.

(taken from N. Ishii et al, PRL 99, 022001 (2007))

-> long range dominated by one-pion exchange.

-> medium range part receives contributions from the exchange of multipions and heavy mesons.

-> short range (r<= 1 fm) : strong repulsive core (quark-gluon structure of the nucleon).

-> two-nucleon data (phase shifts, deuteron binding energy) well reproduced by realistic potentials but...

Nuclear many-body physics

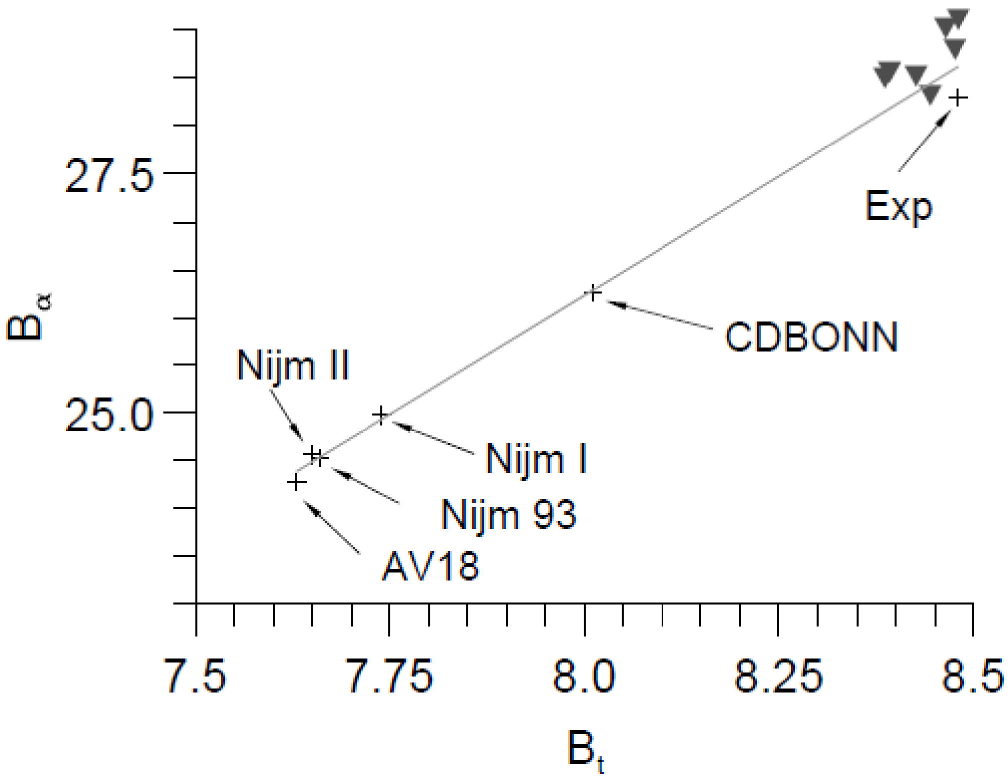
i) More than a sum of two-body interactions.

"intrinsic" three (many)-body forces :

-> nucleon are not point particles (i.e not elementary)

-> some degrees of freedom are neglected e.g Δ -resonance, polarization effects.....

-> many-body forces depend on the two-body interaction



(taken from Nogga et al,
Phys. Rev. Lett. 85, 944-947 (2000))

ii) difficulty associated with strong repulsive core at short distance (high energy)

-> large model space necessary (Shell Model), Hartree-Fock converges slowly (or not at all) with bare interactions...

-> possible solutions : softening of the core by using similarity transformations of the n-n potential (Lee-Suzuky transformation, Similarity Renormalization Group)........

-> **induced many-body forces**

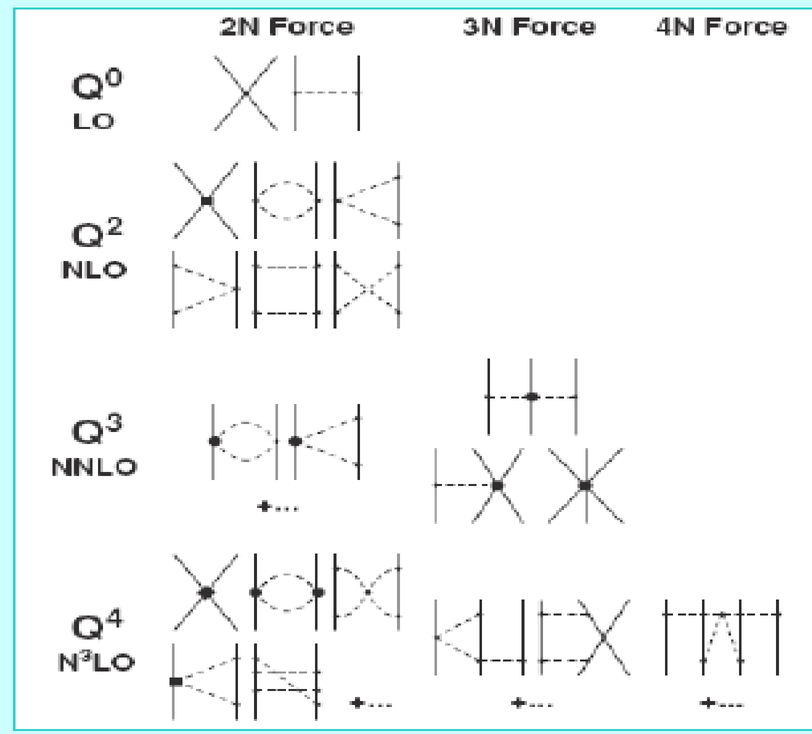
Effective Field Theory

S. Weinberg, Nucl. Phys. B363, 3 (1991).
U. van Kolck, Prog. Phys. Rev. C 49, 2932 (1994).
E. Epelbaum *et al*, Nucl. Phys. A 637, 107 (1998)

$$\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{EFT} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

-> construction of soft interaction.
-> many-body and two-body interactions in a same framework.
-> improvable order by order.

Feynman diagrams



R. Machleidt and D. R. Entem, J. Phys. G 31 (2005) S1235

Effective Field Theory (1/3)

i) Separation of scale :

M_{QCD} ~ 1 GeV (mass of nucleon)
M_{nucl} ~ 100 MeV (typical momentum in a nucleus)
M_{struct} ~ 10 MeV (binding energy of a nucleon in a nucleus)

-> details of physics at short distance (high energy) are irrelevant for low energy physics.

-> in EFT low energy degrees of freedom are explicitly included (high momenta are integrated out).

ii) The Lagrangian / potential consistent with symmetries is expanded as a Taylor Series:

$$V(\vec{p}\,', \vec{p}) = \sum_{i,j} C_{i,j} (\vec{p})^i (\vec{p}\,')^j$$

Effective Field Theory (2/3)

iii) Regularization and renormalization :

-> cut-off Λ (separation between low and high energy physics)

$$V(\vec{p}\,',\vec{p}) => \sum_{i,j} C_{i,j}(\Lambda)(\vec{p})^i(\vec{p}\,')^j$$

-> no dependence on cut-off for observables (for a high enough cut-off), dependence absorbed by coupling constants (fitted with observables).

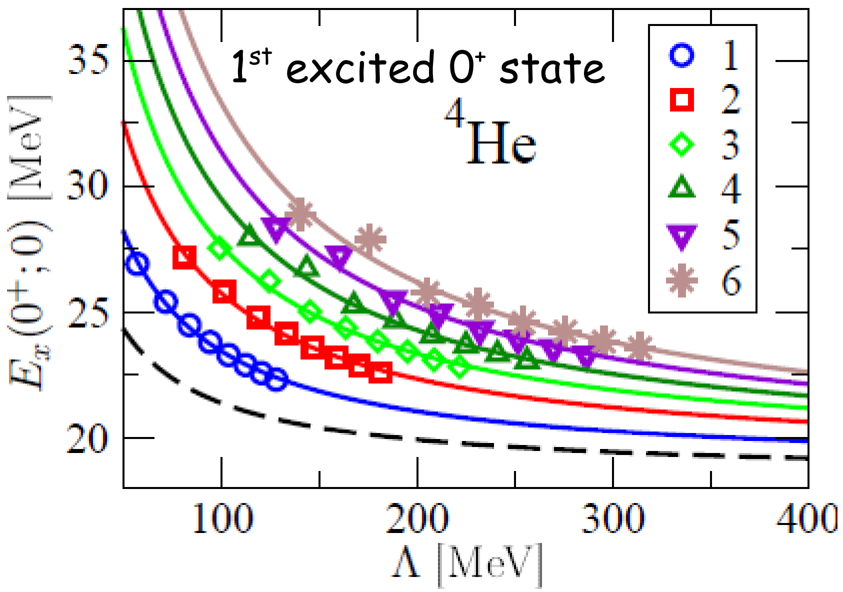
Effective Field Theory (3/3)

iv) Find the power counting ("truncation of the Taylor series"):

-> hierarchy between the different contributions

-> results improvable order by order (Leading Order, Next-to-Leading-Order, Next-to-Next-to-Leading-Order......)

Construction of an effective field theory within the No Core Shell Model



Stetcu et al, PLB653, 2007

-> calculation at **Leading order** : two N-N contact interactions in the $^3S_1$, $^1S_0$ channel and a three-body contact interaction in the 3-nucleon $S_{1/2}$ channel

-> coupling constants fitted to the binding energy of the deuteron, triton and ^{4}He.

Question : How to construct an EFT within a bound many-body model space beyond **Leading-Order** ?

Answer : by trapping the nuclei in a harmonic trap

$$\frac{\Gamma\left(\frac{3}{4}-\frac{\varepsilon}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{\varepsilon}{2}\right)} = -\frac{bk}{2}\cot\delta$$

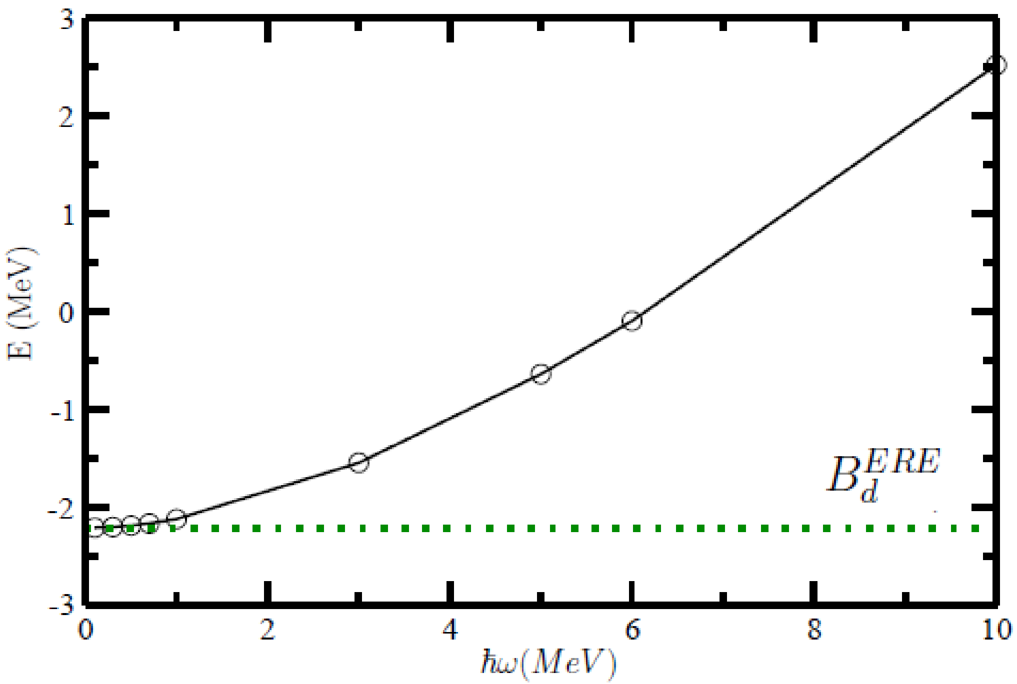
-> relation between the bound state energy and the scattering amplitude (similar to the Lüscher relation used in Lattice QCD)

$$\frac{\Gamma\left(\frac{3}{4}-\frac{\varepsilon}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{\varepsilon}{2}\right)} = -\frac{bk}{2}\cot\delta$$

-> valid in the limit of zero range interaction, r<<b
-> phase shift in the trap distorted with respect to the free space phase shift

How good is the trap to describe continuum physics ?

Energy of a deuteron in a trap



ω should be as small as possible

Deuteron binding energy
from Effective Range Expansion

$$k\cot\delta = -\frac{1}{a_2} + \frac{1}{2} r_2 k^2$$

$$ik + k\cot\delta = 0$$

$$a_t = 5,425 fm \quad r_t = 1.75 fm$$

B_d^{ERE} ~-2,221 MeV

Two nucleons in the 3S1 channel at Next-to-Leading order :

$$\left[\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega r^2 + V(r, r')\right] \Phi_{l,j,m}(r) = E\Phi_{l,j,m}(r)$$

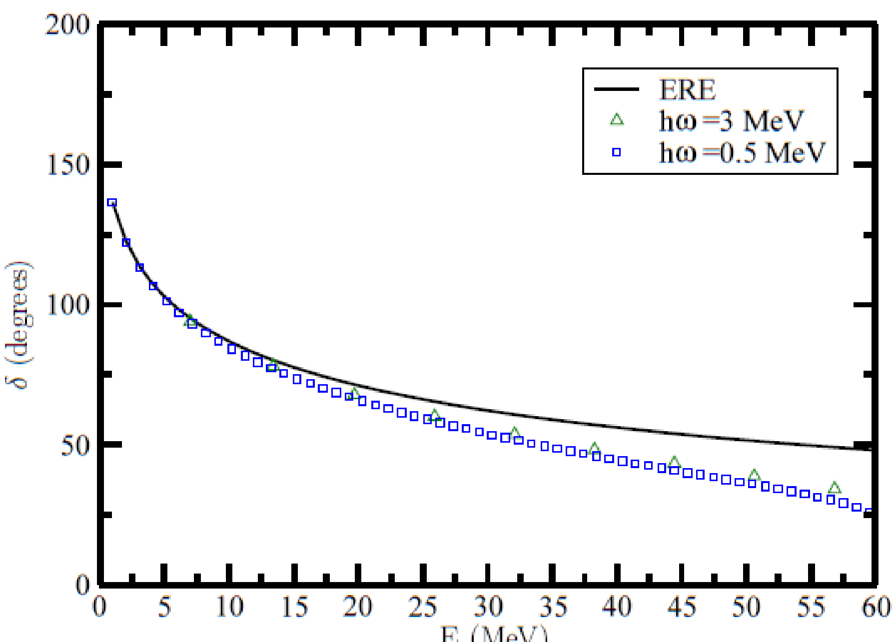
$$\Phi_{l,j,m}(\vec{r}) = \sum_{n}^{n_{max}} a_n \phi_{n,l,j,m}(\vec{r})$$

$$V(\vec{p}', \vec{p}) = c_0(\Lambda(n_{max}, \omega)) + c_2(\Lambda(n_{max}, \omega)) \left(\vec{p}'^2 + \vec{p}^2\right)$$

LO NLO

Coupling constant c_0,c_2 fitted to the ground state (deuteron) and the first excited state in the trap.

$^3S_1$ N-N phase shift in the trap



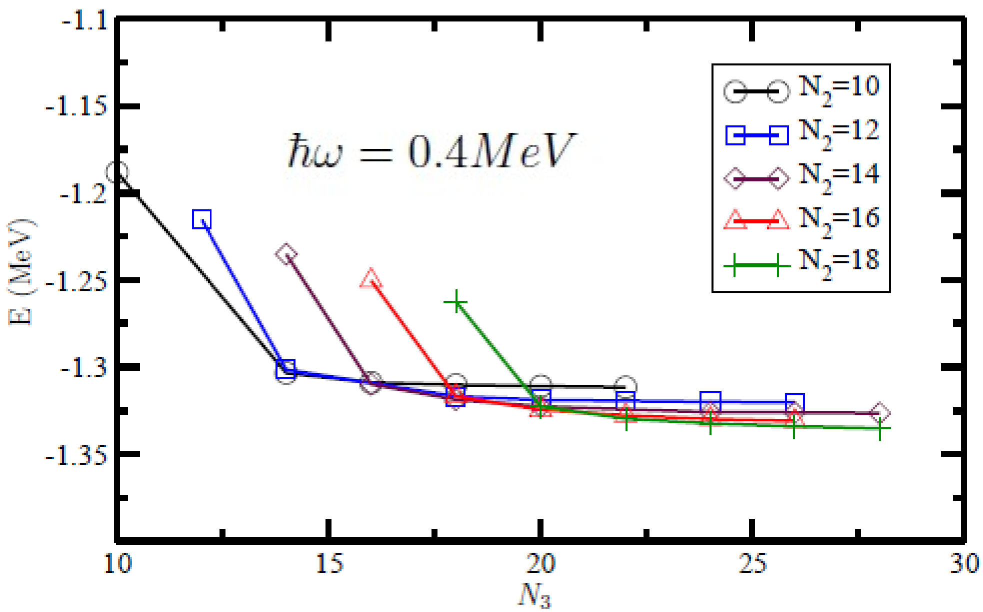
Cutoff defined as

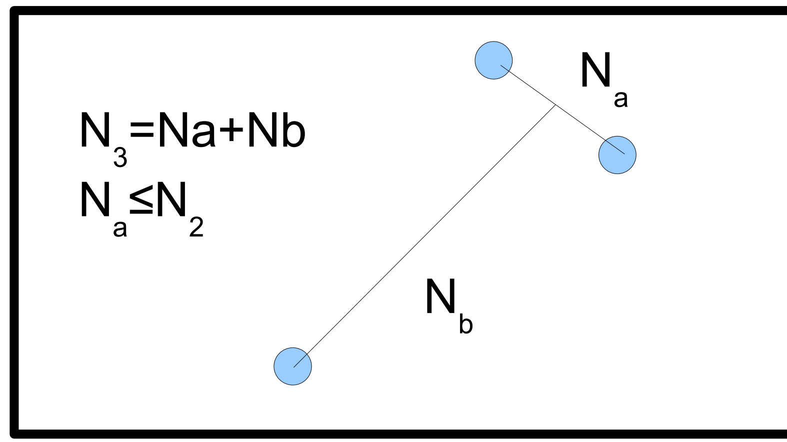
i) n_{max}

ii) $\Lambda = (2n_{max} + l + 3/2)\hbar\omega$

3 nucleons at Leading-Order in the trap coupled to $J^\pi = \frac{3}{2}^+$

for a fixed two-body cutoff (N_2), the size of the model space (N_3) is increased until convergence





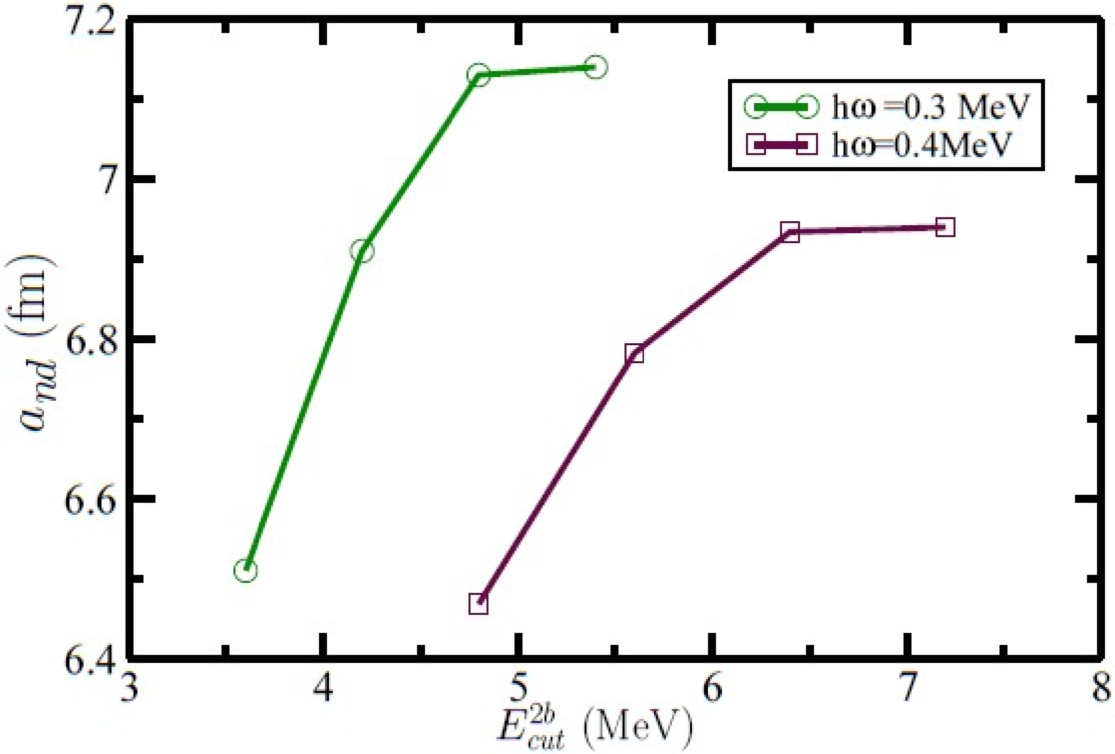
-> convergence of energy as the two-body cutoff N_2 increases

-> as expected no need for a three body force at Leading Order.

For a weak trap, the lowest states coupled to $J^\pi = \frac{3}{2}^+$ correspond to n-d scattering .

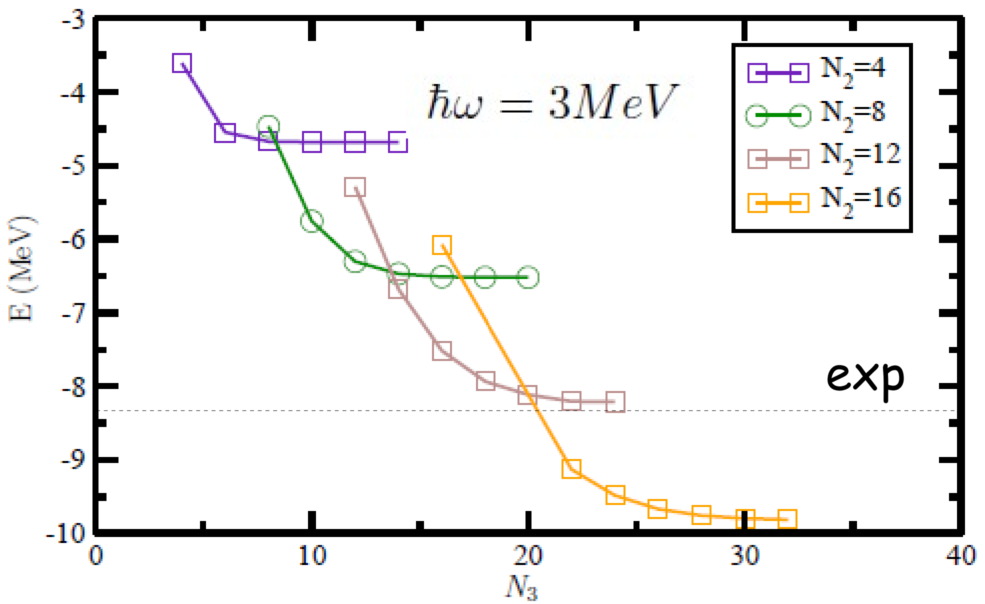
$$\frac{\Gamma\left(\frac{3}{4} - \frac{E_3 - E_d}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_3 - E_d}{2\hbar\omega}\right)} = \frac{b'}{2a_{n-d}} - \frac{r_{n-d} b' k^2}{4} + \ldots.$$

Scattering length of the n-d L=0 channel, at Leading Order



$$a^{exp}_{n-d} = 6.35 \pm 0.10 \text{ fm}$$

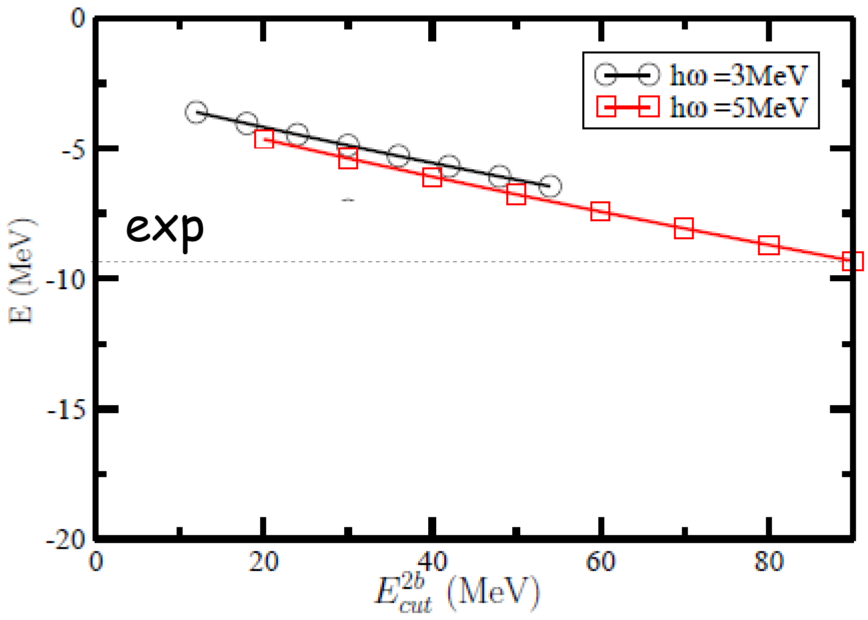
Binding energy of the triton at Leading Order in the trap



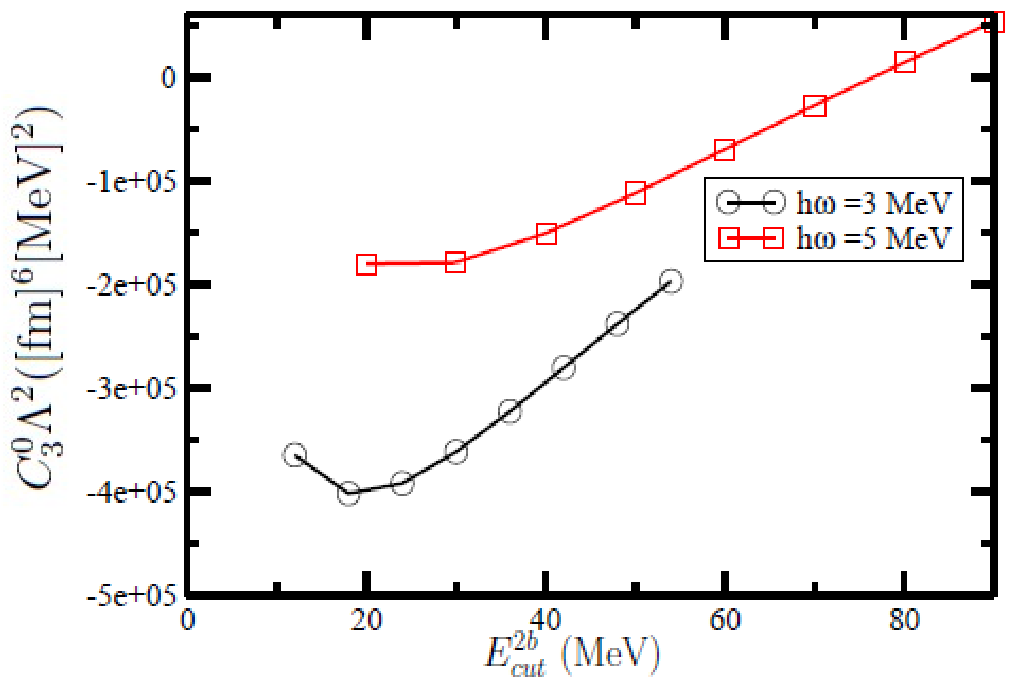
-> the 3 nucleons collapse as the two (three) body cutoff is increased (Efimov effect)

-> need for a three-body force at LO (as in the continuum)

Binding energy of the triton in the trap



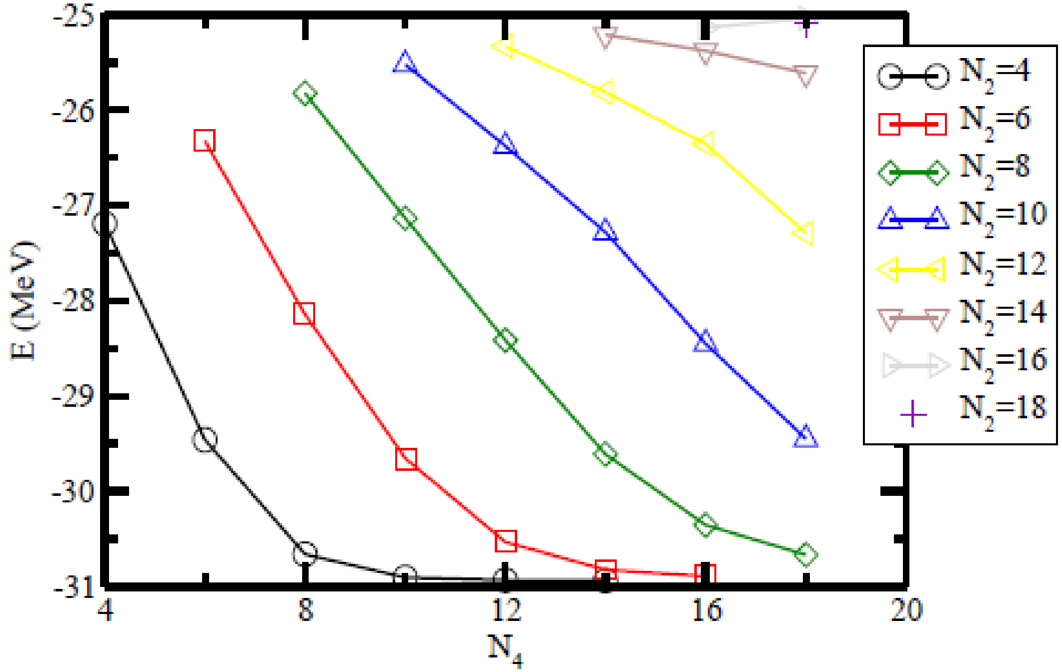
running of the three-body force at LO



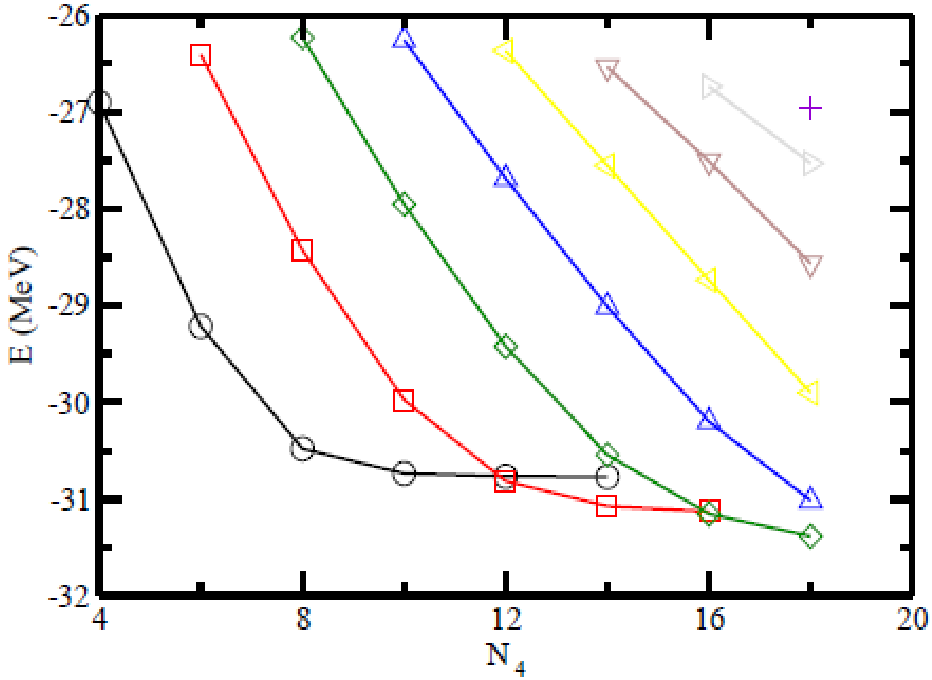
^{4}He g.s in a trap

$$\hbar\omega = 3MeV$$

Leading-Order



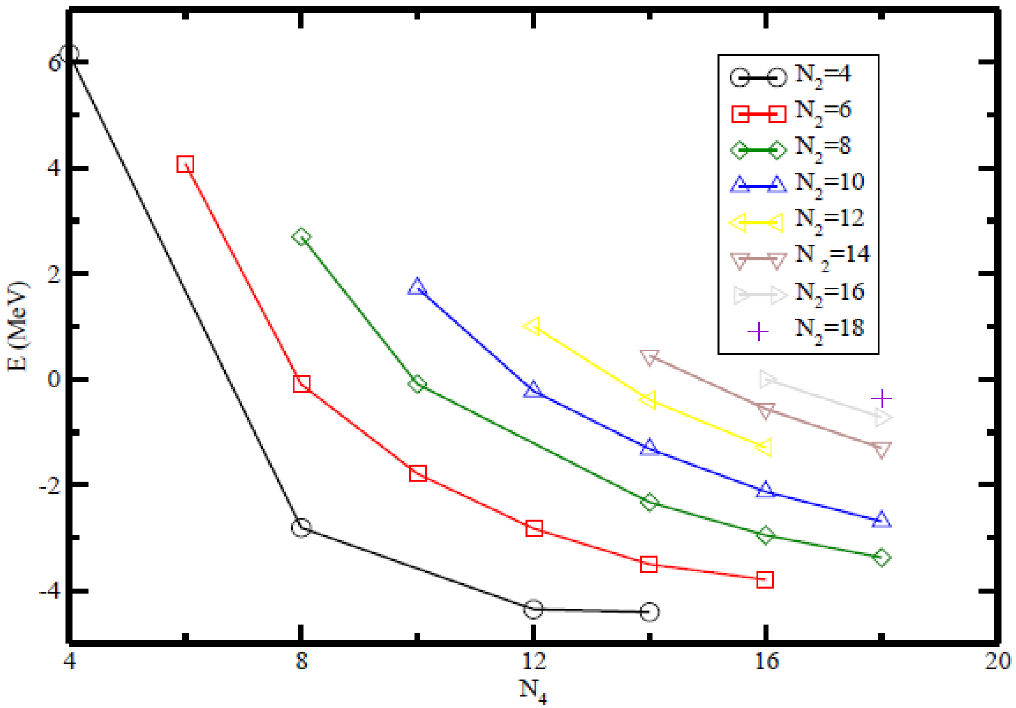
Next-to-Leading-Order



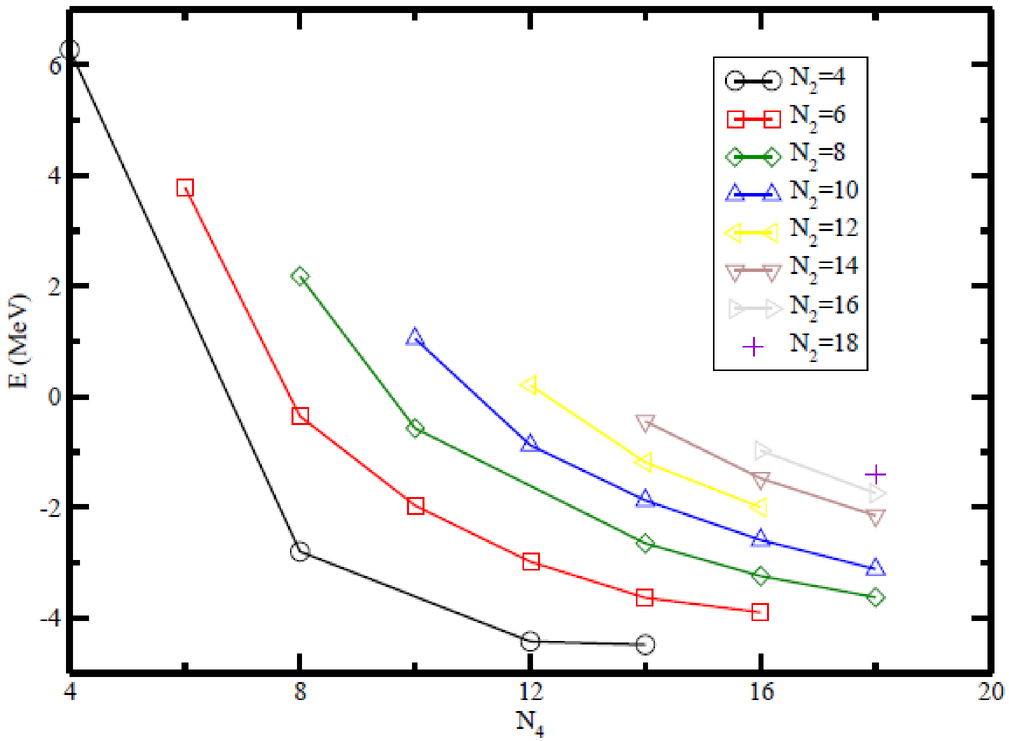
1st excited state in ^{4}He in a trap

$$\hbar\omega = 3MeV$$

Leading-Order



Next-to-Leading-Order

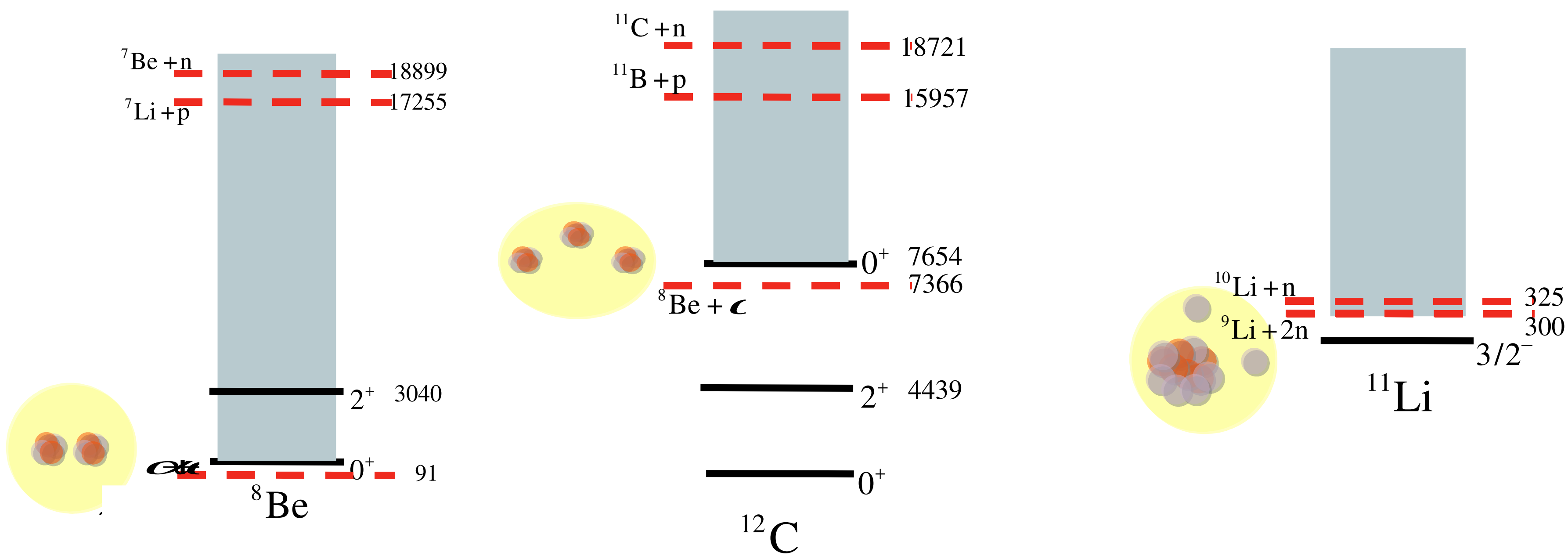


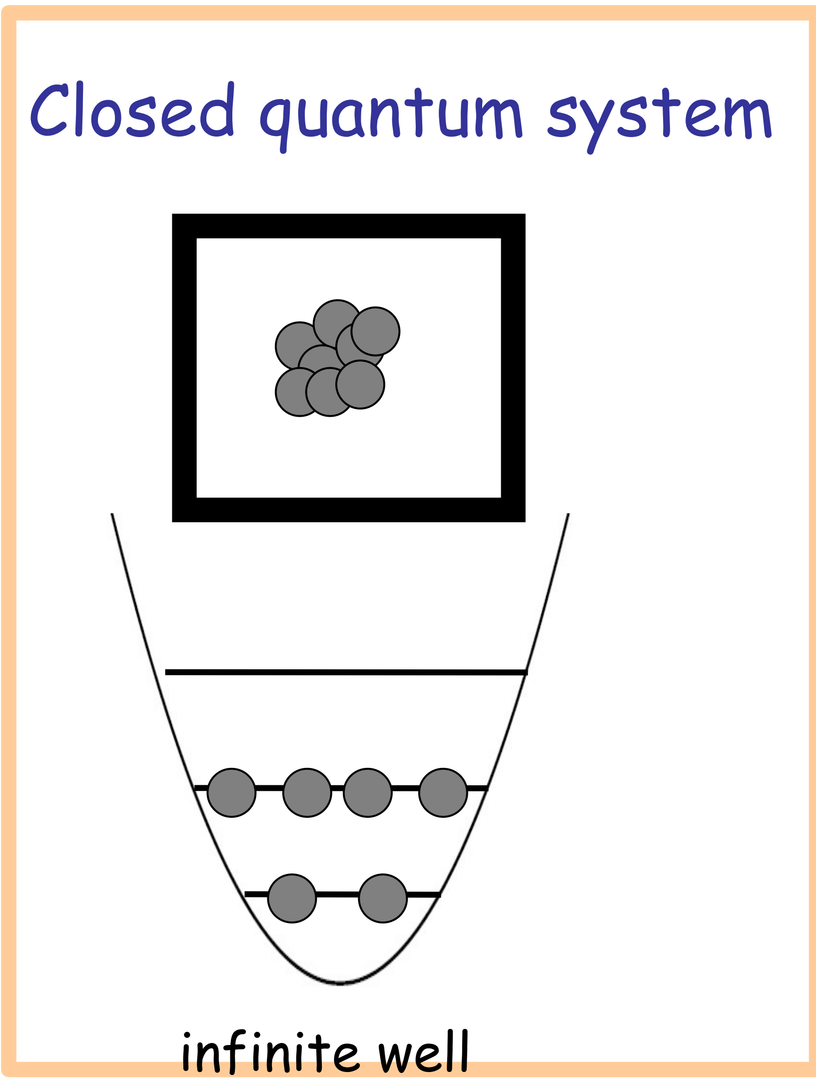
Shell Model for nuclei far from stability

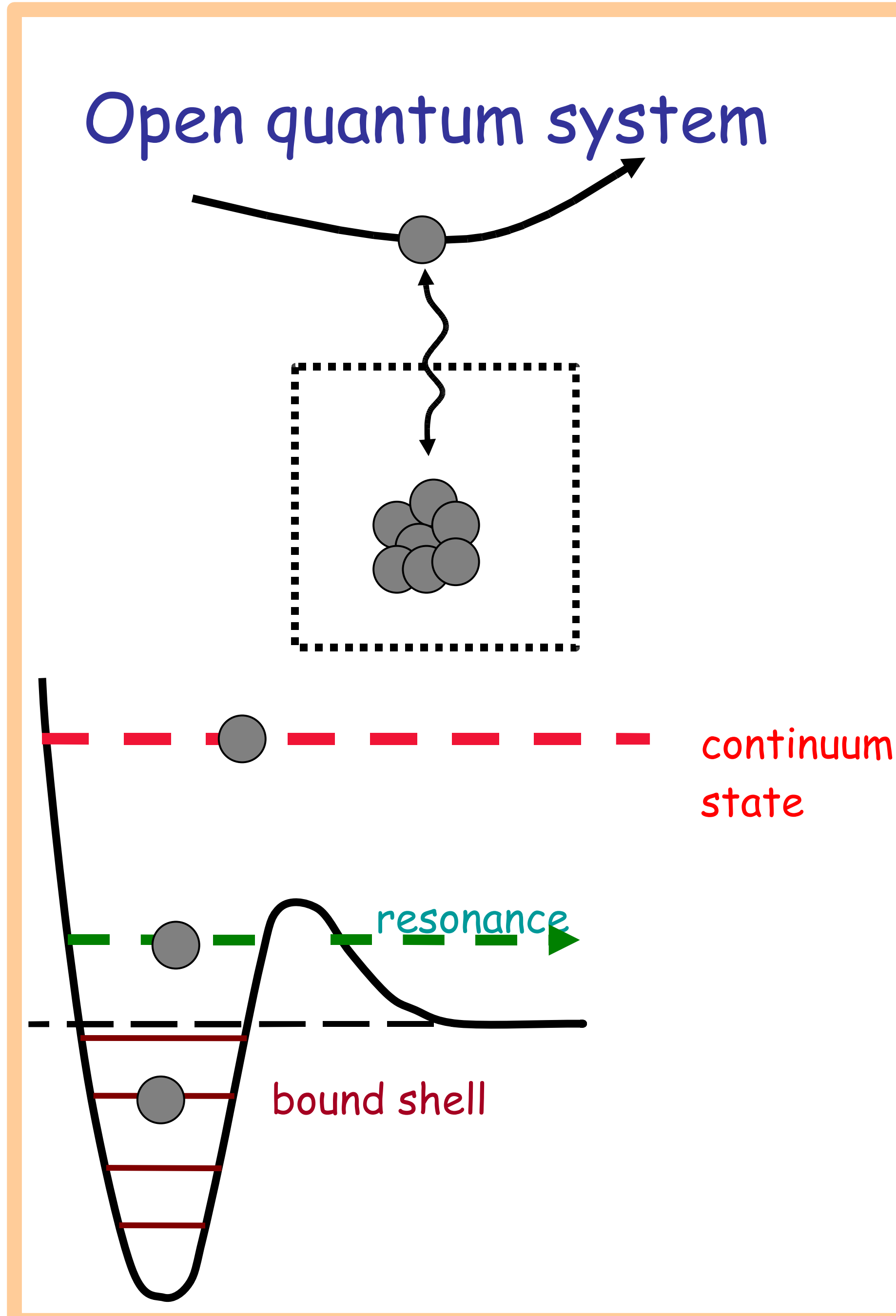
Coupling with the continuum

Cluster states close to the threshold

Halo structure

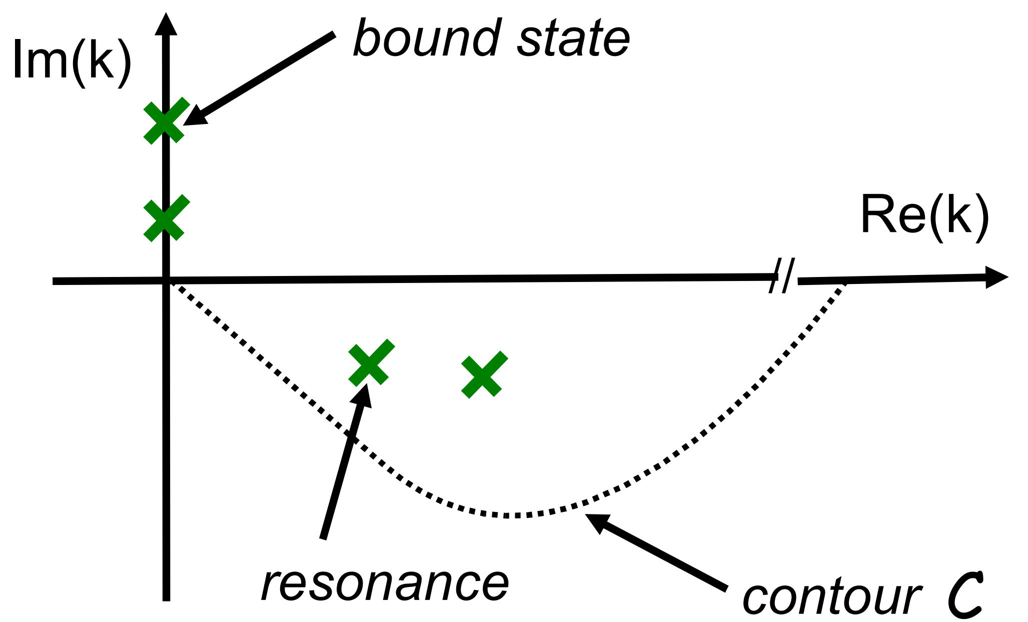



Closed quantum system
infinite well


Open quantum system
continuum
state
resonance
bound shell

Gamow Shell Model

N. Michel et al., Phys. Rev. Lett. 89, 042502 (2002)
N. Michel et al., Phys. Rev. C70, 064311 (2004)
G. Hagen et al, Phys. Rev. C71, 044314 (2005)
J.R et al., PRL. 97, (2006) 110603
N. Michel et al, J Phys G 36, 013101, 2009
K. Tsukiyama et al, PRC 80 051301, 2009



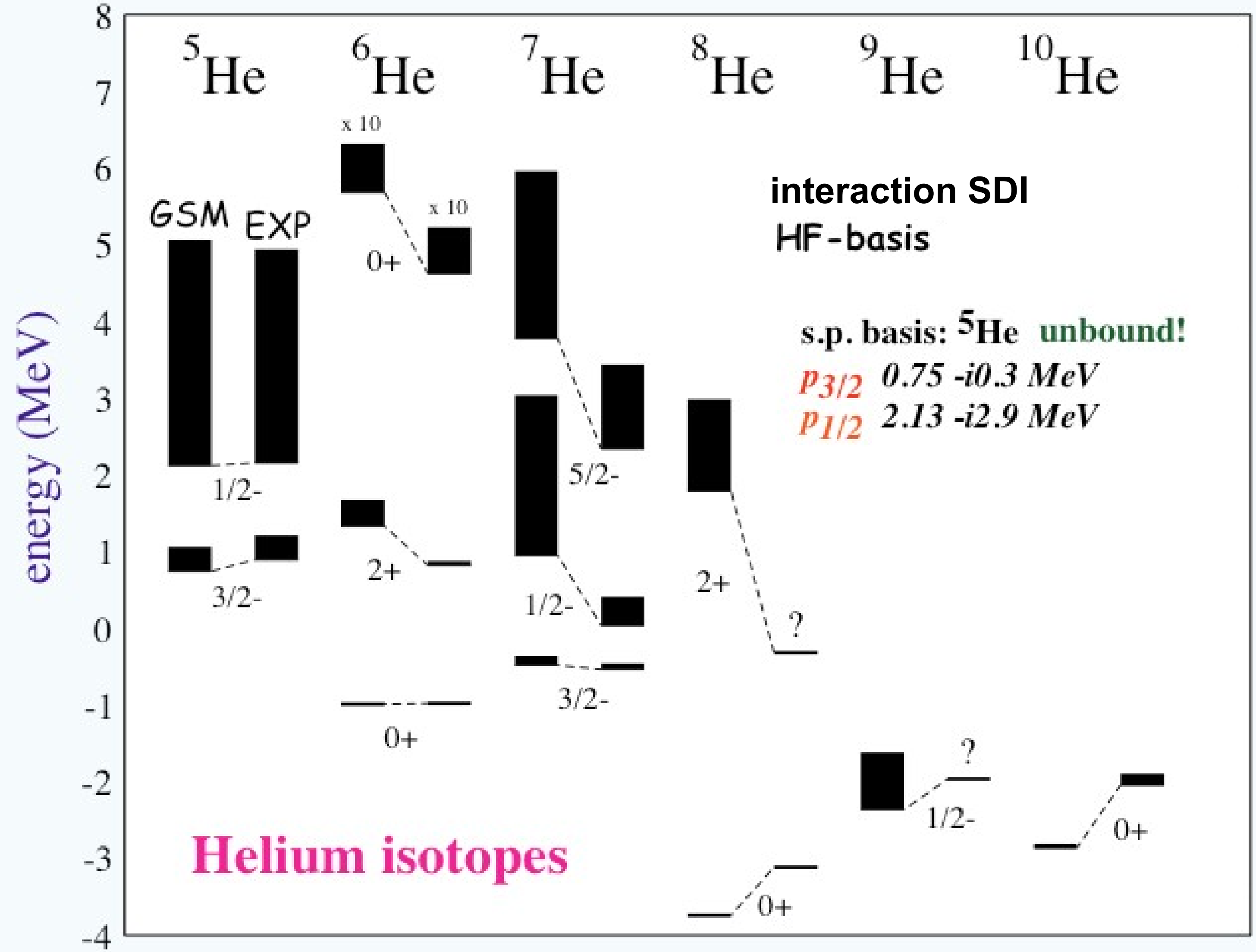
(Berggren completeness relation)

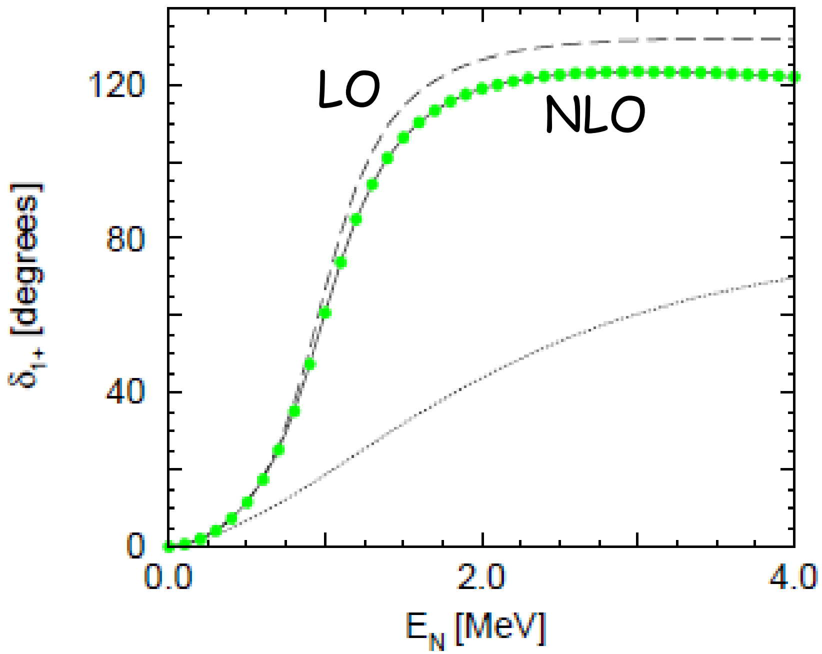
$$\sum_{\text{pole}} |u_n\rangle\langle\tilde{u}_n| + \int_C dk|u_k\rangle\langle\tilde{u}_k| = 1$$

$u(r) \sim H^{+}_{l,\eta}(kr)$ (bound, resonant state)

$u(r) \sim C^{+}H^{+}_{l,\eta}(kr) + C^{-}H^{-}_{l,\eta}(kr)$ (complex-continuum state)

GSM: N. Michel et al., Phys.Rev.Lett. 89, 042502 (2002)





Phase shift for n-alpha scattering in the $p_{3/2}$ partial wave

Effective Field Theory for Halo Nuclei: Shallow p-Wave States, C. Bertulani et al, NPA 712 (2002) 37-58

At Leading Order

-> dimeron potential : $V(k,k') = \dfrac{\vec{k}\cdot\vec{k}'}{(A+BE)}$

-> coupling constants A, B fitted with the scattering length and the effective range

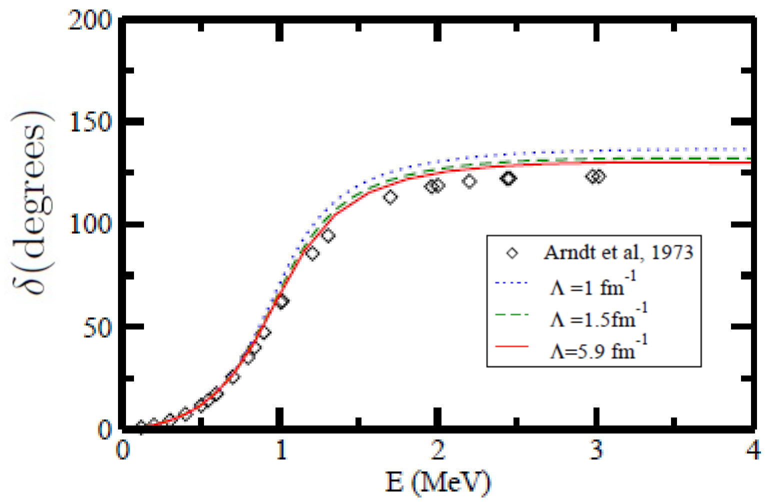
Partial wave $l_\pm$	$a_{l\pm}$ [fm^{1+2l}]	$r_{l\pm}$ [fm^{1-2l}]	$\mathcal{P}_{l\pm}$ [fm^{3-2l}]
0+	2.4641(37)	1.385(41)	–
1−	−13.821(68)	−0.419(16)	–
1+	−62.951(3)	−0.8819(11)	−3.002(62)

-> « pseudo » dimeron potential for Gamow Shell Model calculations (**FOR PRACTICAL REASON**)

$$V(k,k') = \vec{k}\cdot\vec{k'}(C+DE)$$

-> Can the potential be written like that ?

-> numerical proof by checking the cut off (in)dependence



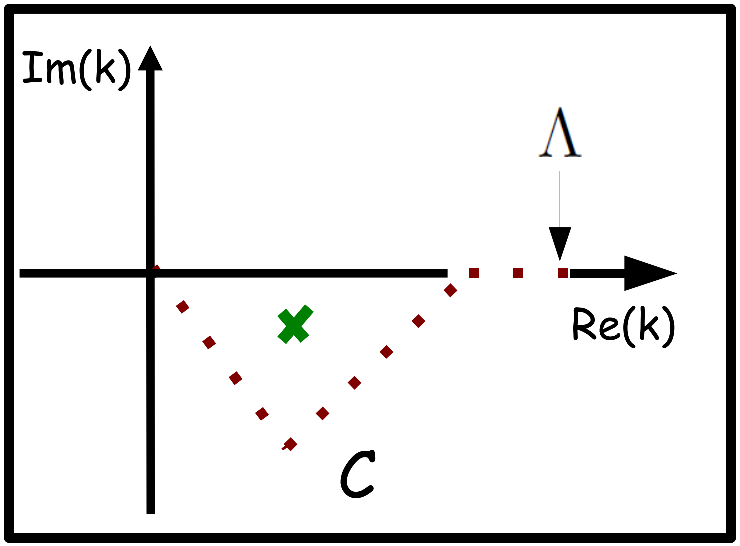
coupling constants fitted with the scattering length and the effective range

Phase shift for n-alpha scattering in the $p_{3/2}$ partial wave

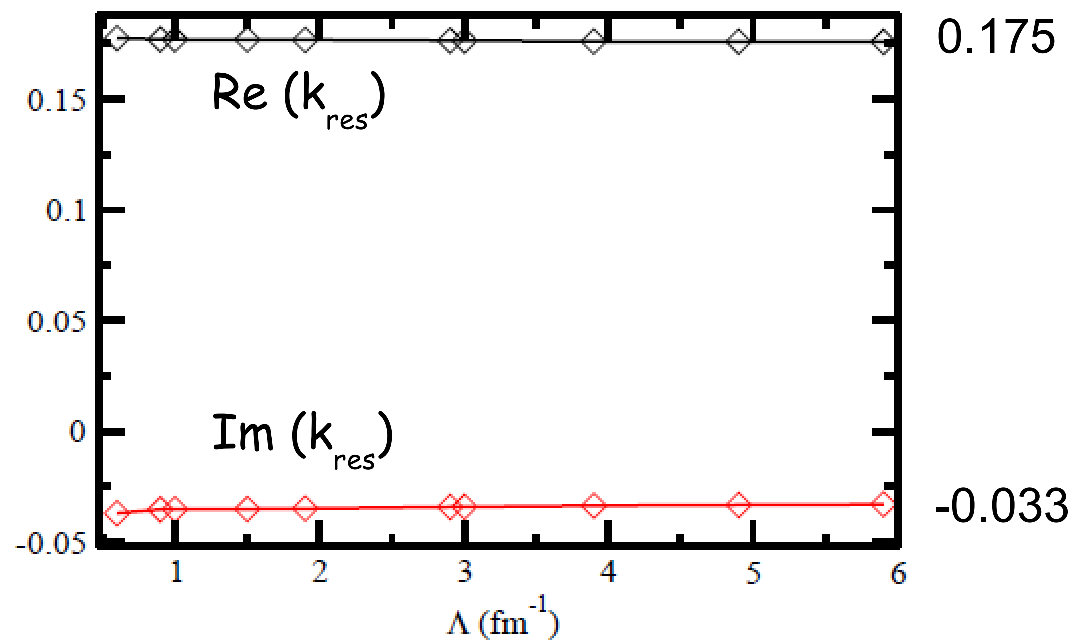
-> cut-off independence of the phase shift !

-> resolution of the Schrödinger equation in the complex k-plane

$$\frac{\hbar^2}{2\mu}k^2\Psi_n(k) + \int_C dk' V(k,k')\Psi_n(k') = E_n\Psi_n(k)$$



Real and imaginary part of the resonance as a function of the cutoff



k^{exp}=(0.18812, -0.0329845)

Next step : include the n-n interaction at LO to describe ^{6}He

Effective Field Theory within a No Core Shell model approach :

-> two-body and many-body forces in a same framework
-> as expected :
- no need for three-body force at Leading Order for the three nucleon system coupled to $J^\pi = \frac{3}{2}^+$
- 3-body force at Leading Order for the triton

->the four-body system needs more investigation

Gamow Shell Model applications for halo nuclei

-> description of $p_{3/2}$ resonance in ^5He with an EFT derived interaction

Future Applications :

-> physics of nuclear few-body systems from input at the QCD scale given by future lattice QCD calculations.
-> description of halo nuclei with the Gamow Shell Model and EFT derived interaction

Collaborators:

B. Barrett, University of Arizona, Tucson

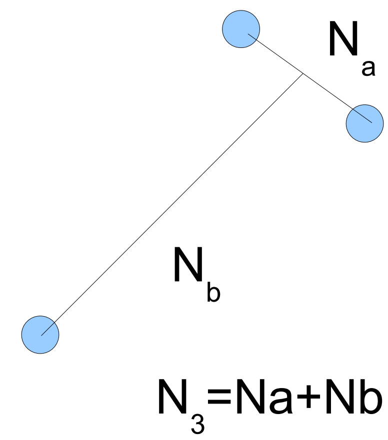
I. Stetcu, University of Washington, Seattle

U. Van Kolck, University of Arizona, Tucson

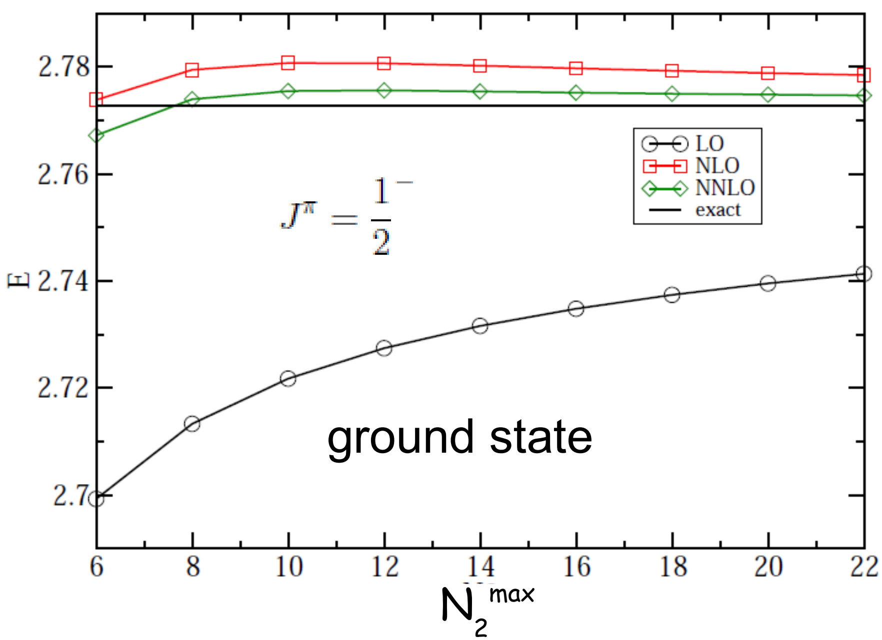
M. Birse, The University of Manchester, UK

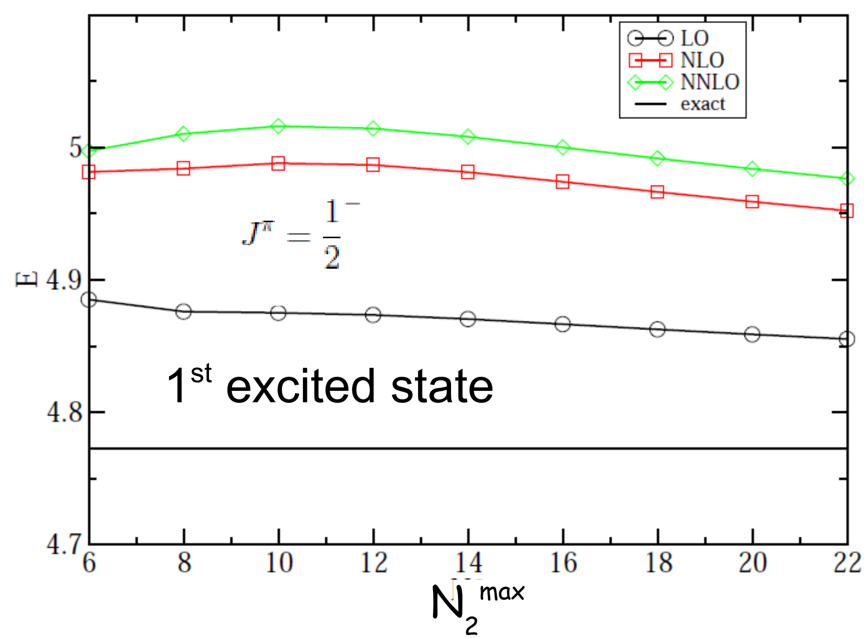
Three fermions in a trap

-> two-body interaction defined by cutoff N_2^{max} and three body model space defined by cutoff N_3^{max}



A) cut off *a la* Shell Model : two body cut-off N2 is fixed by N3.



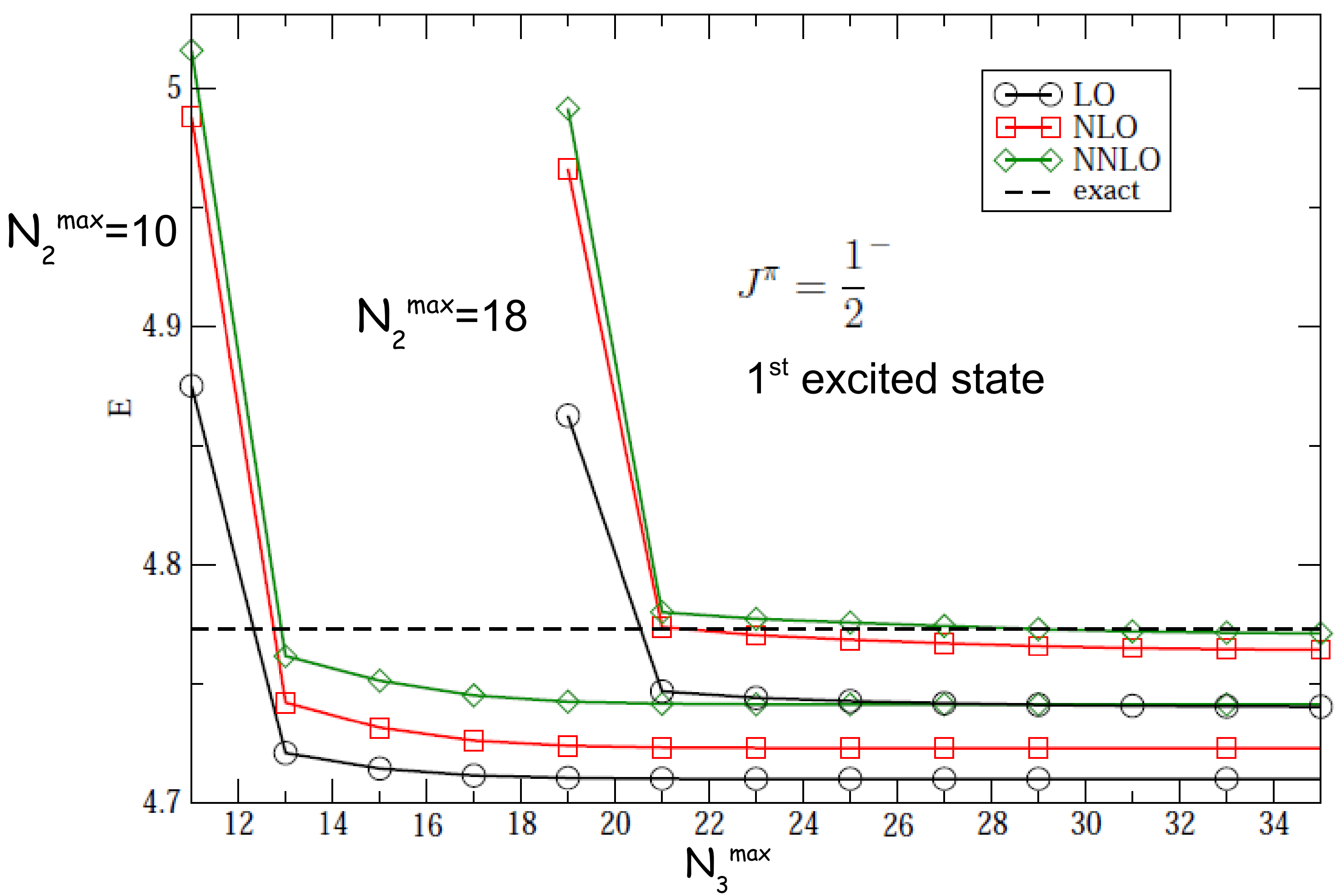


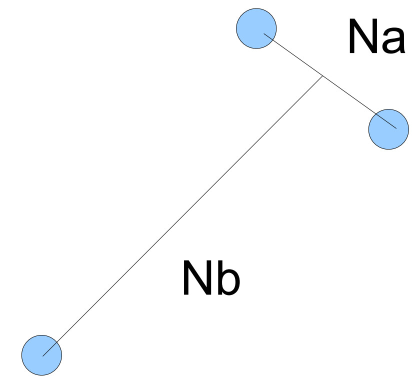
→ Problem !!! : NLO , NNLO further away from exact value than LO for some states

Three fermions in a trap

B) Solution :

-> for a fixed two-body cut-off N_2^{max} the three body cut-off N_3^{max} is increased until convergence (completeness is reached).





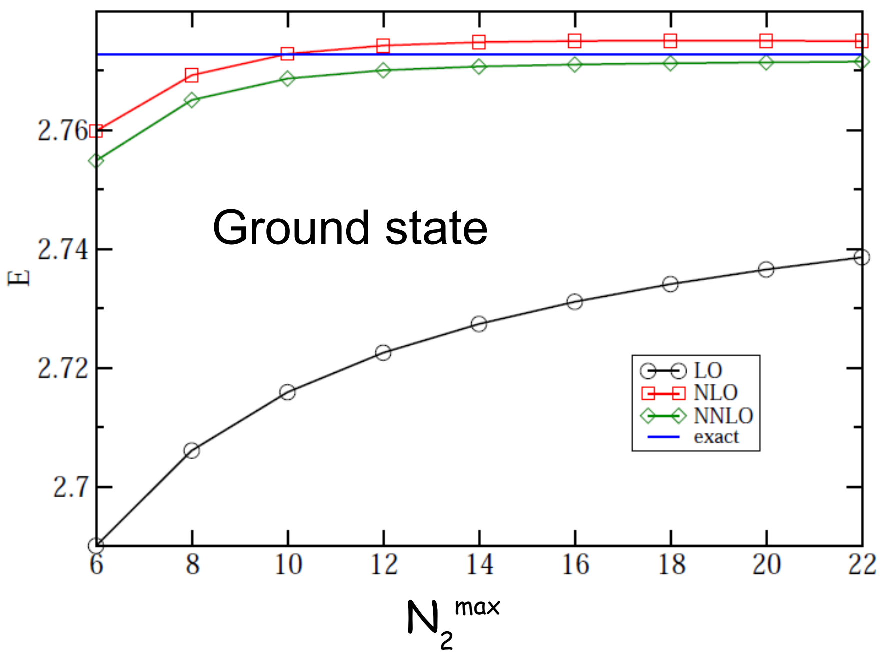
N3=Na+Nb

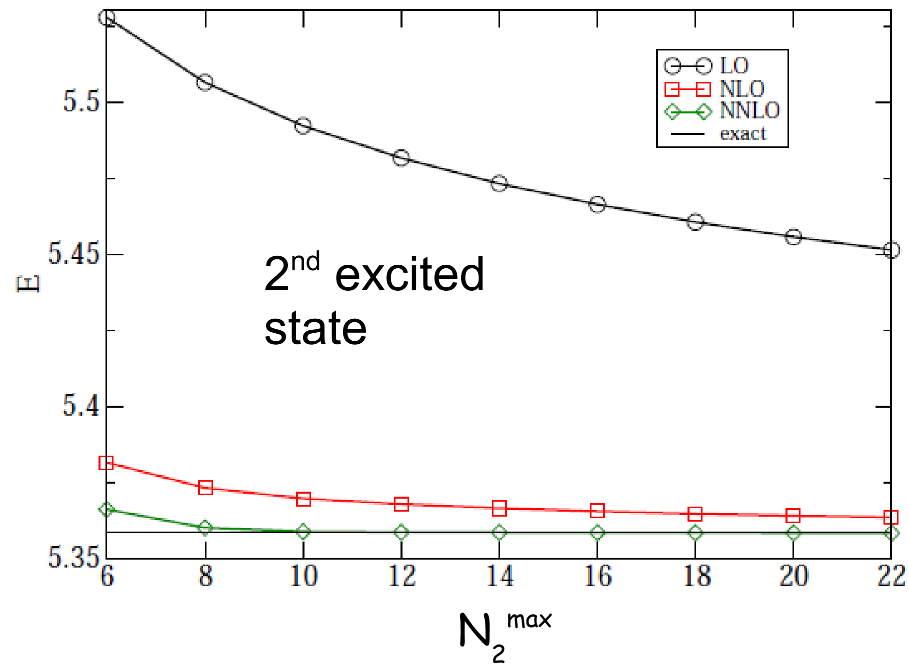
->no restriction on N3

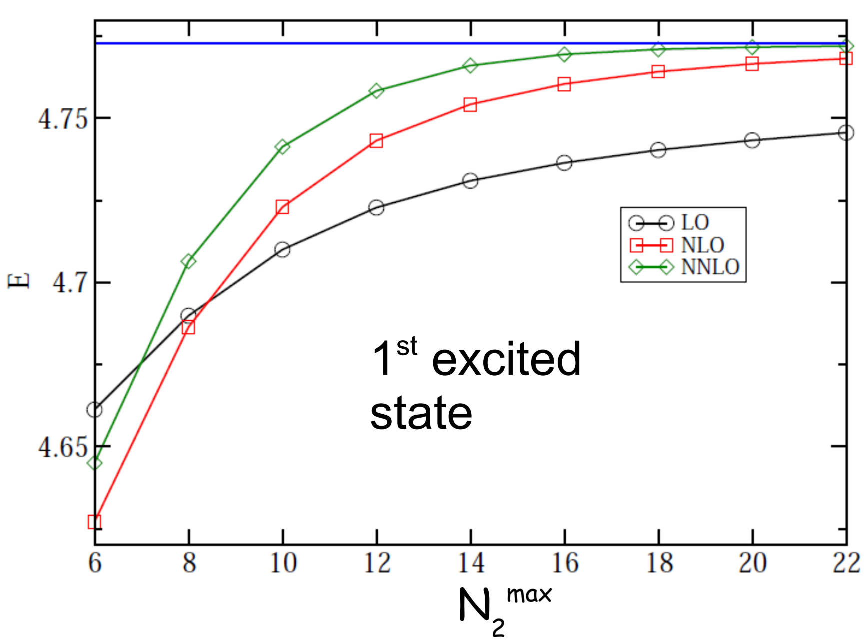
-> for $N_a > N_2^{max}$ the interaction is "switched off"

-> correct ordering of the different orders, faster convergence

J. R, I.Stetcu, B. Barrett, U. Van Kolck, M. Birse, PRA 82, 2010







-> convergence increases as more corrections are considered