Universality of short-range correlations in nuclei

International workshop on
"The limits of existence of light nuclei"
25-30 October, 2010
ECT* Trento, Italy

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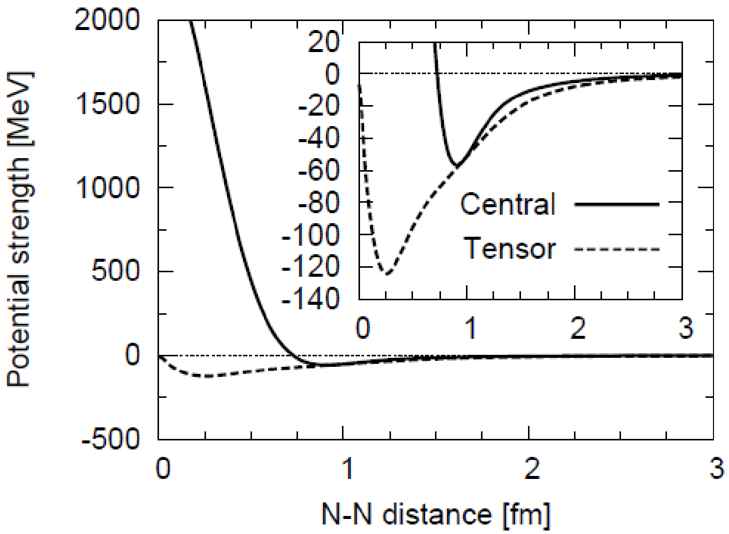
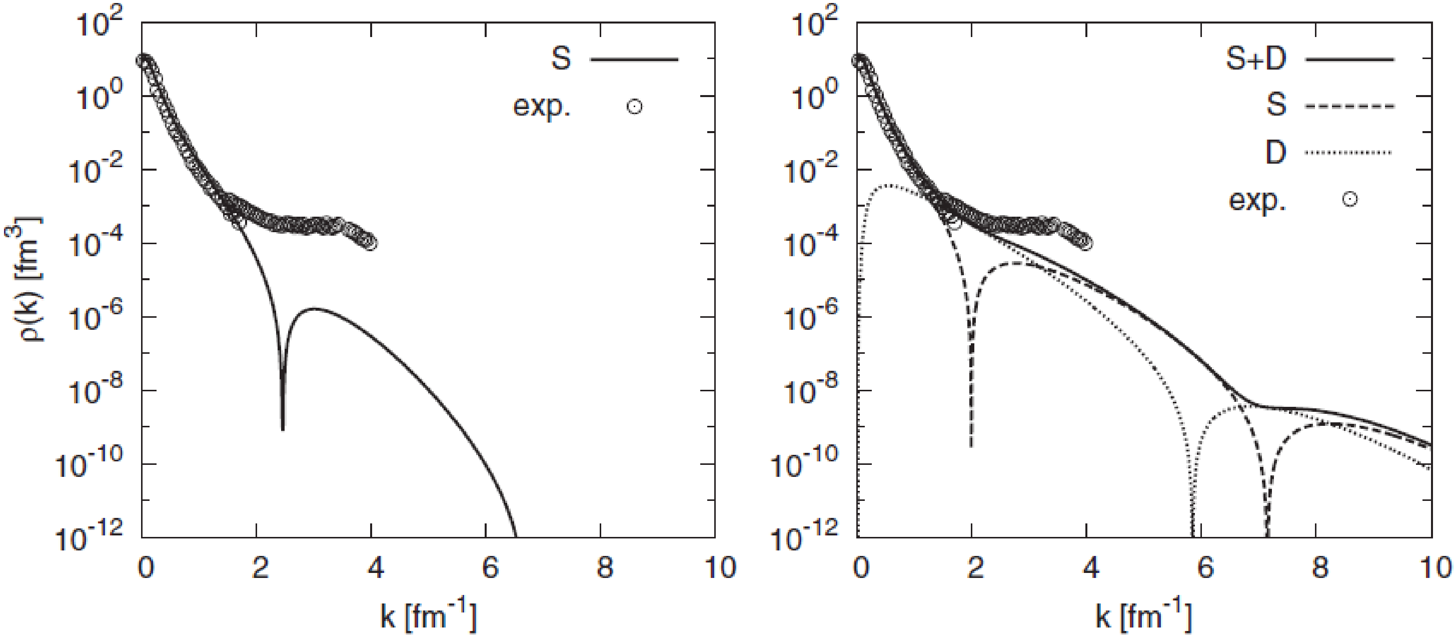
Introduction

- “Realistic” nucleon-nucleon force
  - N-N phase shift
  - Deuteron properties
- Coordinate space
  - Short-range repulsion
  - Strong tensor component
- Momentum space
  - High momentum
  - Off diagonal matrix element

Argonne potential (V8’)



High momentum component


Only central

Short-range + tensor

- Momentum distribution: Observable
- (e, e’) Measurement at JLAB

- Saturation property of nuclear matter
- Important information for effective interactions

Purpose of the study

- Extract information on short-range and tensor correlations

- Highly correlated many-body states
  - Two-, Three-, and Four-body systems
    - ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{4}He* (0_2^+)
  - One-body densities
  - Two-body densities

Outline

- Our approach
  - Variational calculation with explicitly correlated basis
  - Correlated Gaussian and global vector
  - Stochastic variational method
- Results
  - One-body densities
  - Two-body densities
- Summary

Variational calculation for many-body systems

Hamiltonian

$$H=\sum_{i=1}^{A}T_i-T_{\rm cm}+\sum_{i<j}^{A}v_{ij}+\sum_{i<j<k}^{A}v_{ijk}$$

$$v_{12}=V_c(r)+V_{\rm Coul.}(r)P_{1\pi}P_{2\pi}+V_t(r)S_{12}+V_b(r)\boldsymbol{L}\cdot\boldsymbol{S}$$

AV8' interaction: central, tensor, spin-orbit

Generalized eigenvalue problem

$$\Psi_{JM_J}=\sum_{i=1}^{K}c_i\Psi(\alpha_i)$$

$$\sum_{j=1}^{K}(H_{ij}-EB_{ij})c_j=0\quad(i=1,\ldots,K)$$

$$\begin{pmatrix}H_{ij}\\B_{ij}\end{pmatrix}=\langle\Psi(\alpha_i)|\begin{pmatrix}H\\1\end{pmatrix}|\Psi(\alpha_j)\rangle$$

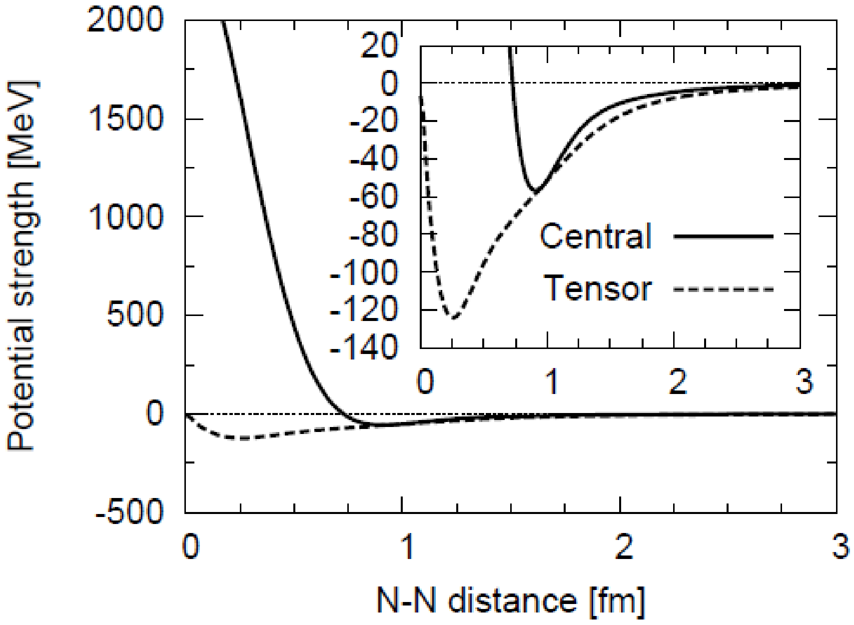
Basis function

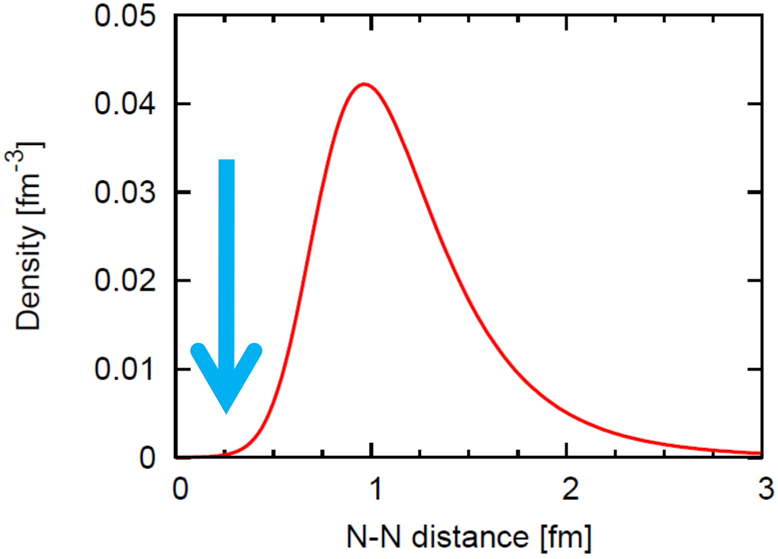
$$\Psi_{(LS)JM_JTM_T}=\mathcal{A}\left\{\left[\psi_L^{(\rm space)}\psi_S^{(\rm spin)}\right]_{JM_J}\psi_{TM_T}^{(\rm isospin)}\right\}$$

$$\psi_{SM_S}^{(\rm spin)}=\left|[\cdots[[[\tfrac{1}{2}\tfrac{1}{2}]S_{12}\tfrac{1}{2}]S_{123}]\cdots]SM_S\right\rangle$$

Hard to obtain a precise solution using a realistic interaction

^{3}E





- Short-range repulsion

  → A superposition of many basis states

- Strong tensor component

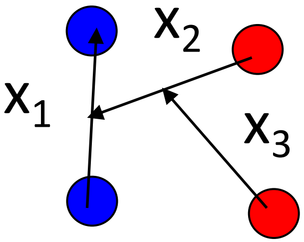
  → Angular momentum coupling

Explicitly correlated basis function

Correlated Gaussian

$$\exp\left(-\frac{1}{2}ar^2\right) \to \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right) = \exp\left(-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right)$$

$$\exp\left(A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right) \sim \sum_n (\boldsymbol{x}_i\cdot\boldsymbol{x}_j)^n \sim \sum_{\ell=n,n-2,\dots} [\mathcal{Y}_\ell(\boldsymbol{x}_i)\mathcal{Y}_\ell(\boldsymbol{x}_j)]_{00}$$



Global vector

$$r^l Y_{lm}(\hat{\boldsymbol{r}}) \equiv \mathcal{Y}_{lm}(\boldsymbol{r}) \to \mathcal{Y}_{LM_L}(\tilde{u}\boldsymbol{x}) = \mathcal{Y}_{LM_L}\left(\sum_{i=1}^{A-1} u_i\boldsymbol{x}_i\right)$$

$$\mathcal{Y}_{LM_L}(u_1\boldsymbol{x}_1+u_2\boldsymbol{x}_2) = \sum_{\ell=0}^{L}\sqrt{\frac{4\pi(2L+1)!}{(2\ell+1)!(2L-2\ell+1)!}}\; u_1^{\ell}u_2^{L-\ell}[\mathcal{Y}_\ell(\boldsymbol{x}_1)\mathcal{Y}_{L-\ell}(\boldsymbol{x}_2)]_{LM_L}$$

Global Vector Representation (GVR)

Parity $(-1)^{L_1+L_2}$

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

-> Formulation for N-body system

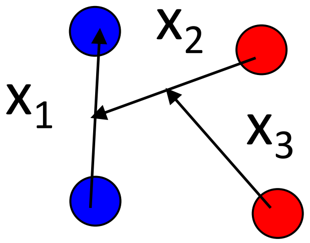
Advantages of GVR

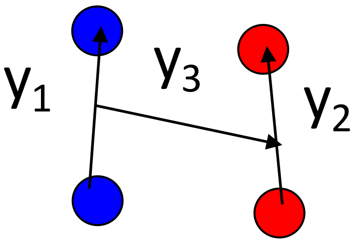
General formulation for N-body system
Matrix elements -> analytically obtained

Variational parameters A, u_1, u_2 → Stochastic variational method (SVM)

- No need to specify intermediate angular momenta.
  - Just specify total angular momentum L
- Nice property for coordinate transformation
  - Antisymmetrization, rearrangement channels

$$F_{(L_1L_2)LM}(u_1, u_2, A, x) = \exp\left(-\frac{1}{2}\tilde{x}Ax\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}x)\mathcal{Y}_{L_2}(\widetilde{u_2}x)]_{LM}$$





$$y = Tx \quad \Longrightarrow \quad \tilde{y}By = \tilde{x}\tilde{T}BTx \qquad \tilde{v}y = \widetilde{\tilde{T}v}x$$

Algorithm of the SVM

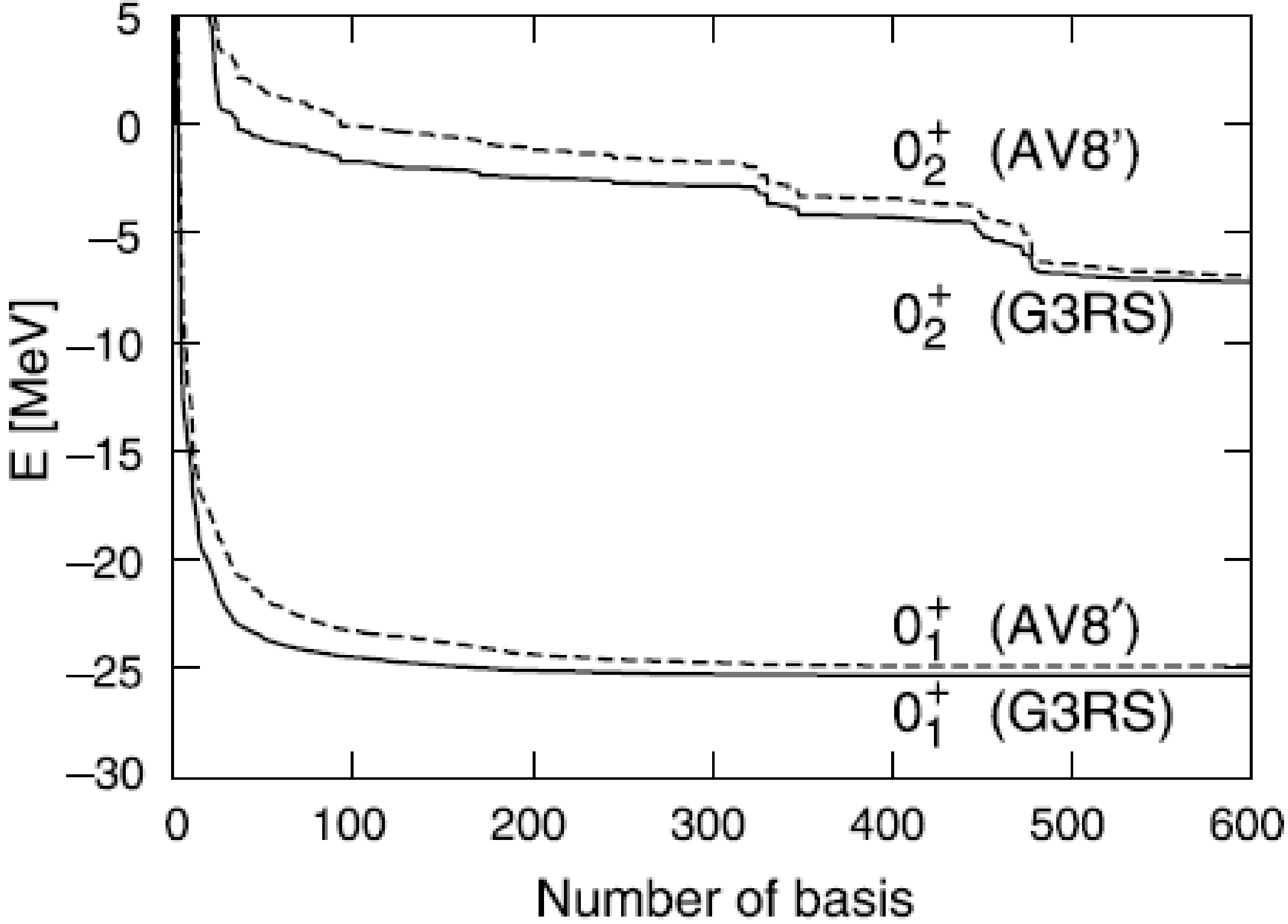
Possibility of the stochastic optimization

1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. **Generate** $(A_k^1, A_k^2, \cdots\cdots, A_k^m)$ **randomly**
2. **Get the eigenvalues** $(E_k^1, E_k^2, \cdots\cdots, E_k^m)$
3. **Select** A_k^n corresponding to the lowest E_k^n and **Include** it in a basis set
4. **k → k+1**

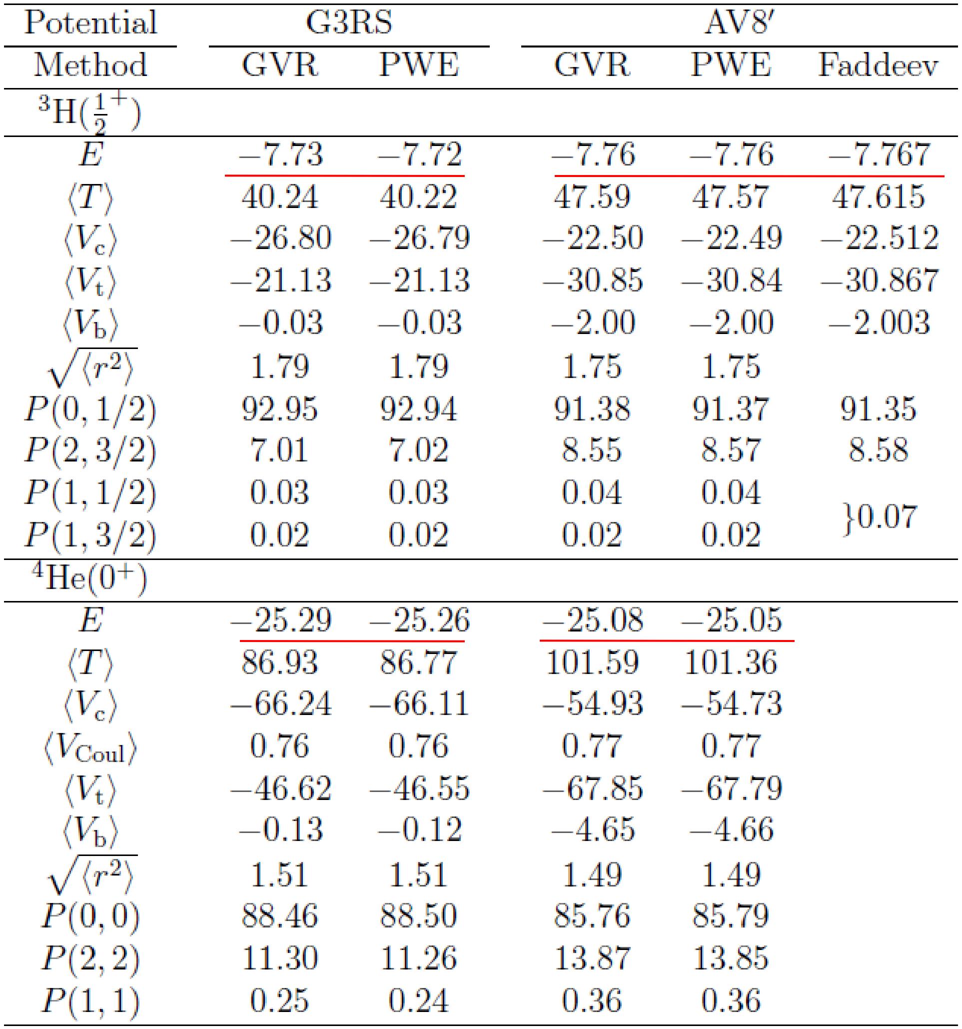
Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

Energy convergence for ^{4}He



Gaussian Expansion Method: ～6000 basis states

Test of GVR

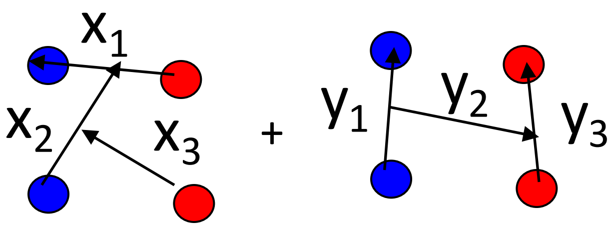


Potential	G3RS		AV8′		
Method	GVR	PWE	GVR	PWE	Faddeev
$^3\mathrm{H}(\frac{1}{2}^+)$					
E	−7.73	−7.72	−7.76	−7.76	−7.767
$\langle T\rangle$	40.24	40.22	47.59	47.57	47.615
$\langle V_\mathrm{c}\rangle$	−26.80	−26.79	−22.50	−22.49	−22.512
$\langle V_\mathrm{t}\rangle$	−21.13	−21.13	−30.85	−30.84	−30.867
$\langle V_\mathrm{b}\rangle$	−0.03	−0.03	−2.00	−2.00	−2.003
$\sqrt{\langle r^2\rangle}$	1.79	1.79	1.75	1.75	
$P(0,1/2)$	92.95	92.94	91.38	91.37	91.35
$P(2,3/2)$	7.01	7.02	8.55	8.57	8.58
$P(1,1/2)$	0.03	0.03	0.04	0.04	}0.07
$P(1,3/2)$	0.02	0.02	0.02	0.02	
$^4\mathrm{He}(0^+)$					
E	−25.29	−25.26	−25.08	−25.05	
$\langle T\rangle$	86.93	86.77	101.59	101.36	
$\langle V_\mathrm{c}\rangle$	−66.24	−66.11	−54.93	−54.73	
$\langle V_\mathrm{Coul}\rangle$	0.76	0.76	0.77	0.77	
$\langle V_\mathrm{t}\rangle$	−46.62	−46.55	−67.85	−67.79	
$\langle V_\mathrm{b}\rangle$	−0.13	−0.12	−4.65	−4.66	
$\sqrt{\langle r^2\rangle}$	1.51	1.51	1.49	1.49	
$P(0,0)$	88.46	88.50	85.76	85.79	
$P(2,2)$	11.30	11.26	13.87	13.85	
$P(1,1)$	0.25	0.24	0.36	0.36	

Comparison with Partial Wave Expansion (PWE)

$$\exp\left(-\frac{1}{2}a_1\boldsymbol{x}_1^2-\frac{1}{2}a_2\boldsymbol{x}_2^2\cdots\right)$$
$$\times[[[\mathcal{Y}_{\ell_1}(\boldsymbol{x}_1)\mathcal{Y}_{\ell_2}(\boldsymbol{x}_2)]_{L_{12}}\mathcal{Y}_{\ell_3}(\boldsymbol{x}_3)]_{L_{123}}\cdots]$$

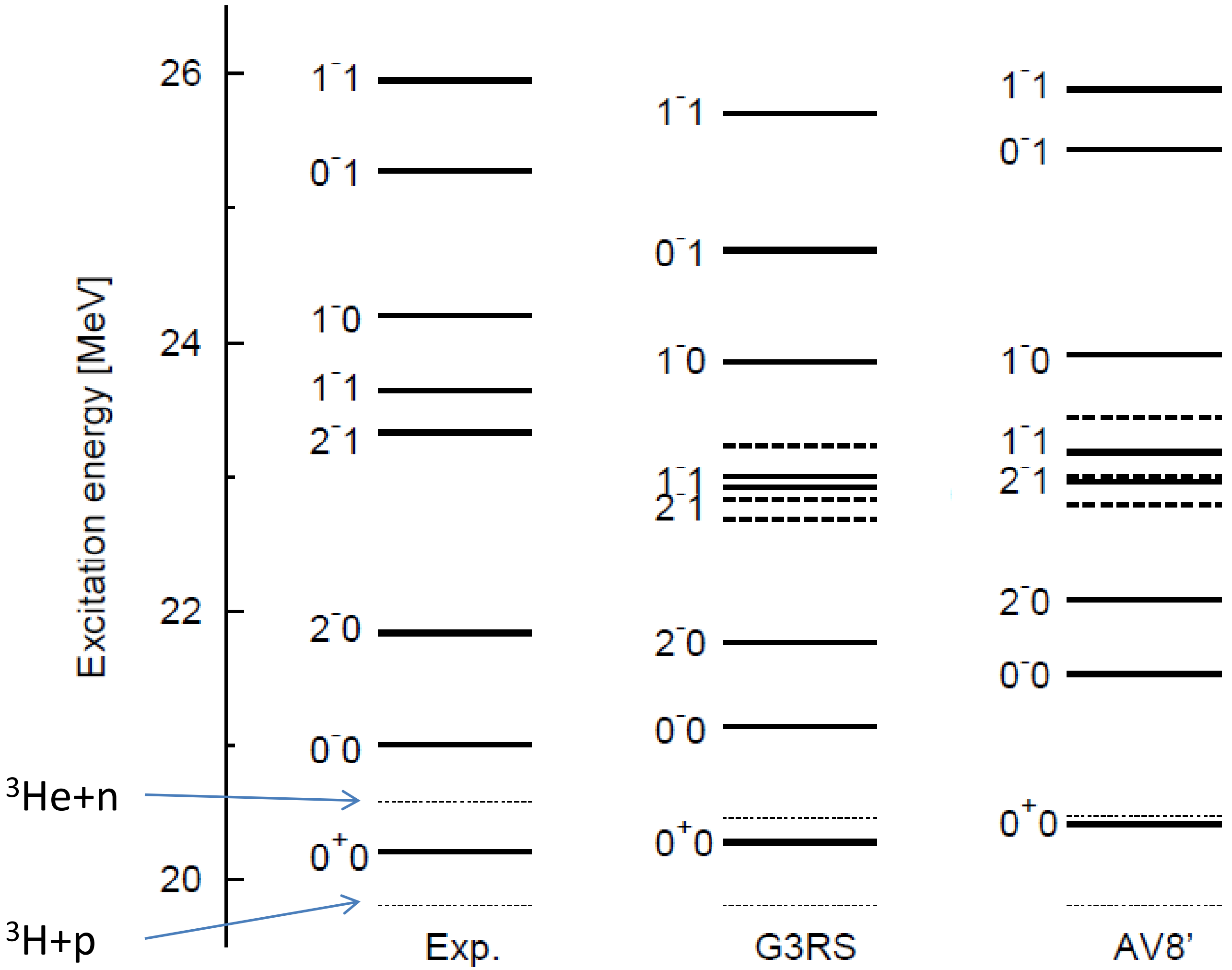
→ combine rearrangement channels



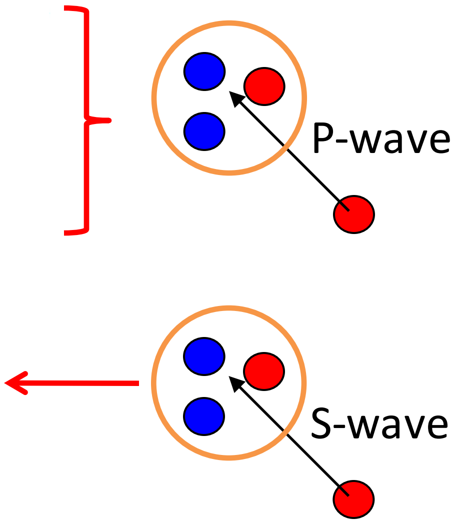
Ground state energy agrees with the other precise methods within 60 keV.
H. Kamada et al., PRC64, 044001 (2001)

Y. Suzuki, W. H., M. Orabi, K. Arai, Few-Body Systems **42**, 33 (2008).

^{4}He spectrum



^{3}H+p, ^{3}He+n cluster structure appear
W. H. and Y. Suzuki, PRC78, 034305(2008)



Good agreement with experiment

How to extract correlated information

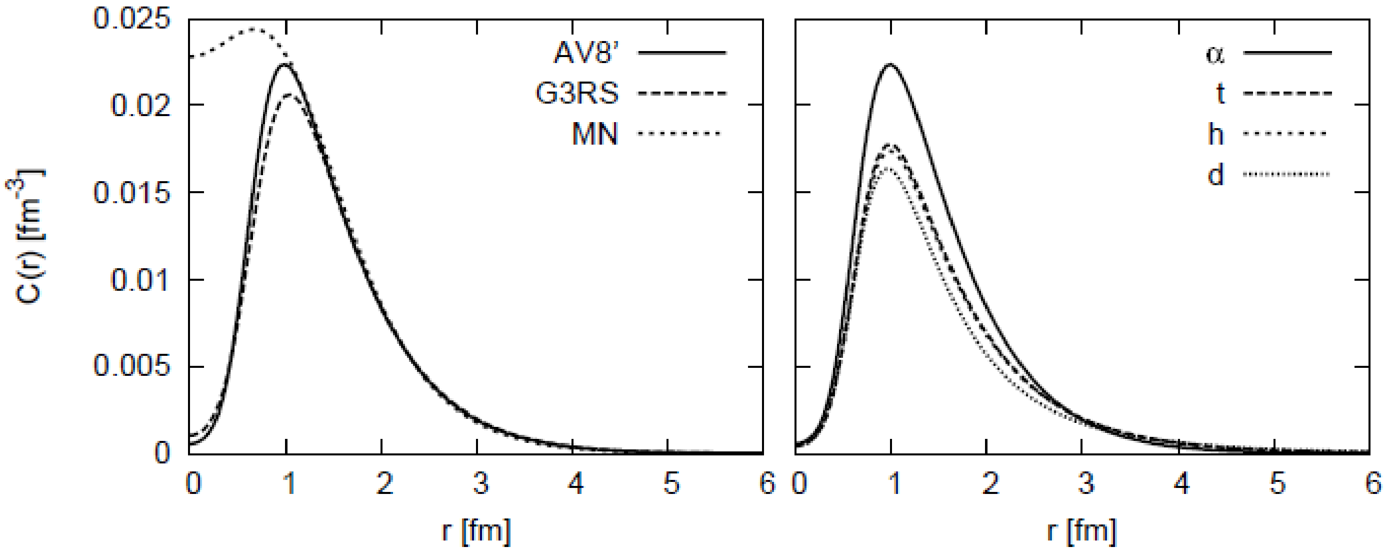
- Antisymmetrized many-body states Φ
  - Two-body: d
  - Three-body: t, h
  - Four-body: α, α* (0_2^+)

- *A*-body density: all information on correlation
  - Too much information
    - Position or momentum vectors: *A*
    - Spin-isospin possibilities: 4**A*
  - Two-body correlation

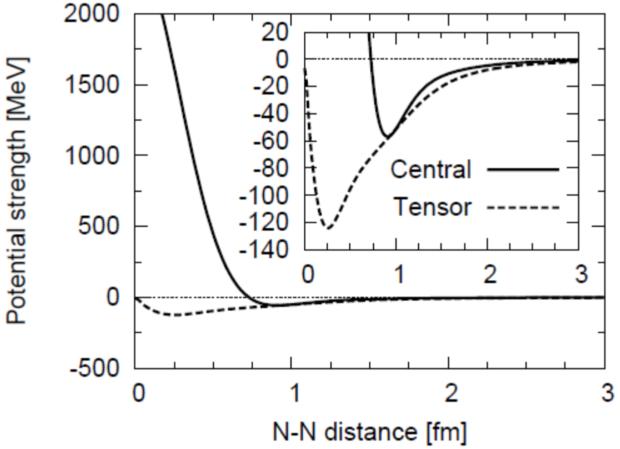
    -> integrate over A-2 particle degrees of freedom

Two-particle correlations in nuclei

Correlation function : $$C(r) = \frac{1}{4\pi r^2} \langle \Phi | \, \delta^3(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r) \, | \Phi \rangle$$



AV8’: central, tensor, spin-orbit
G3RS: softer than AV8’
MN: central only



Effect of the short range repulsion

Size of the systems

Kinetic densities

Correlation function:
(Momentum space)

$$C(k) = \frac{1}{4\pi k^2} \langle\Phi|\, \delta^3\left(\frac{1}{2}|\boldsymbol{k}_1 - \boldsymbol{k}_2| - k\right) |\Phi\rangle$$

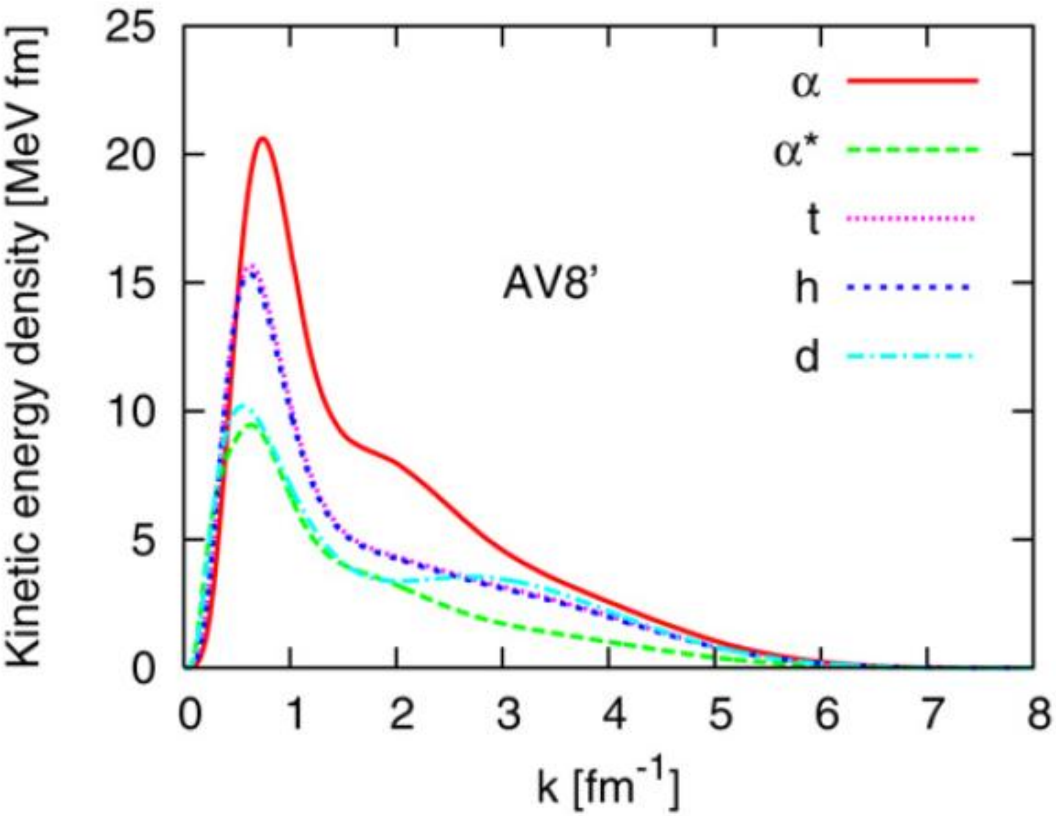
Kinetic energy density:

$$(\hbar^2/m)k^4C(k)$$

Total kinetic energy

$$(N-1)\frac{\hbar^2}{m}\int_0^\infty dk\, k^4C(k)$$

Tensor force
Short-range repulsion



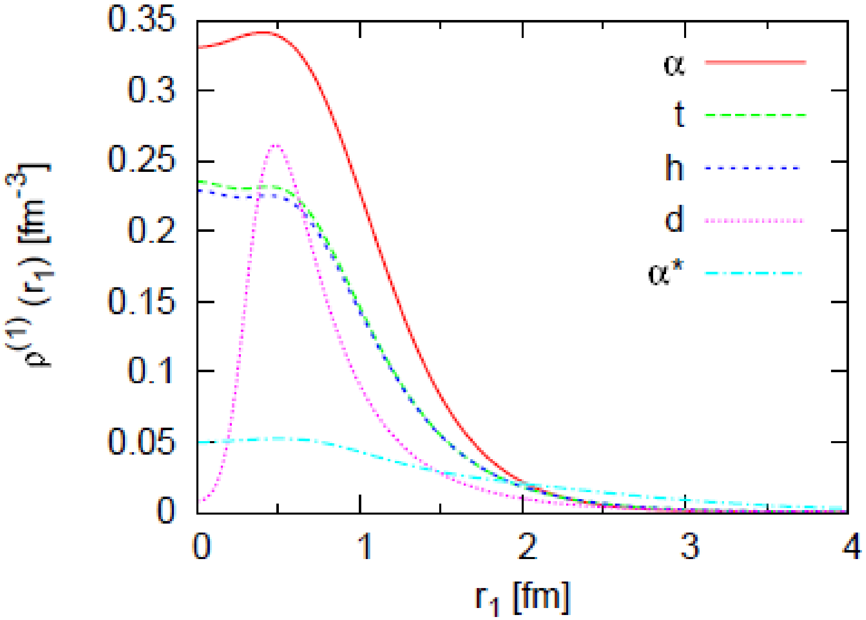
One-body densities

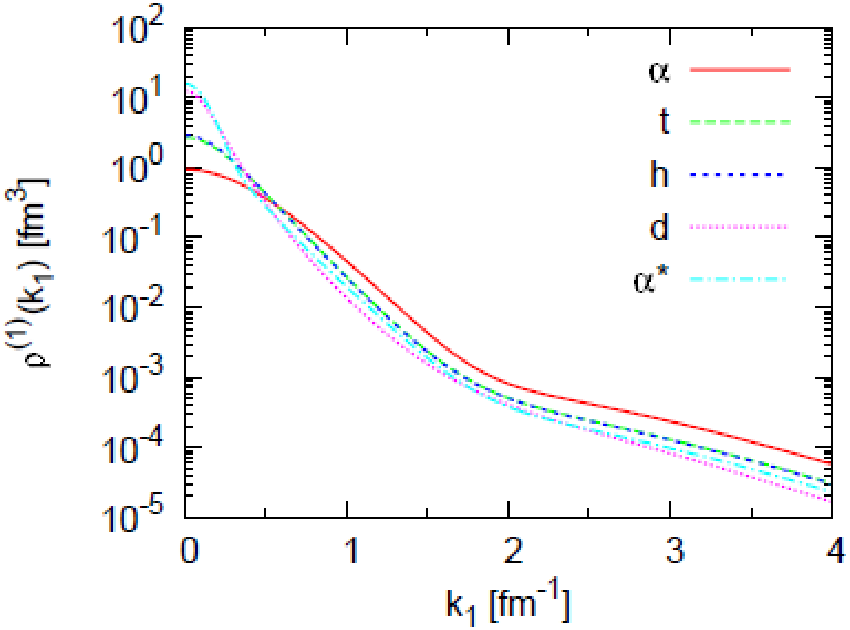
Coordinate space

$$\rho^{(1)}(\boldsymbol{r}_1) = \langle\Phi|\sum_{i=1}^{A}\delta^3(\boldsymbol{r}_i - \boldsymbol{r}_1)\,|\Phi\rangle$$

Momentum space

$$\rho^{(1)}(\boldsymbol{k}_1) = \langle\Phi|\sum_{i=1}^{A}\delta^3(\boldsymbol{k}_i - \boldsymbol{k}_1)\,|\Phi\rangle$$





Size of the system
Different distributions

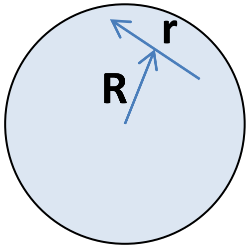
Size of the system
High momentum component

Two-body densities

Two-body density

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_1)\delta^3(\boldsymbol{r}_j-\boldsymbol{r}_2)\underline{\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}}\,|\Phi\rangle$$

Spin (isospin) projector



Two-body density in relative coordinate

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle$$

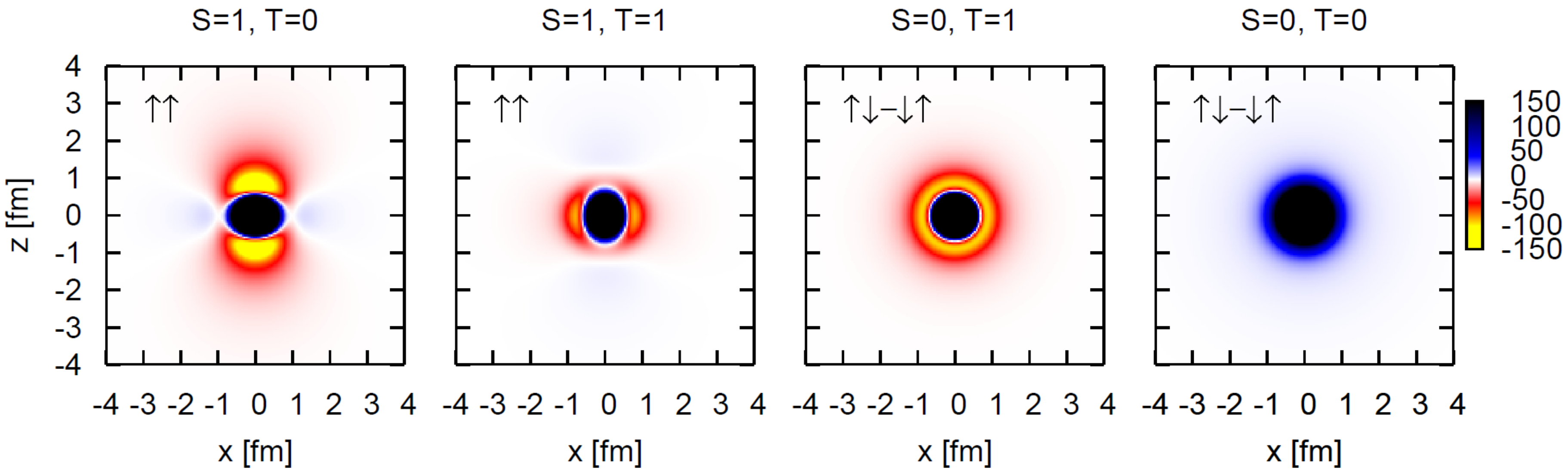
$$\boxed{\begin{aligned}\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r})&=\int d\boldsymbol{R}\,\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R})\\&=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle\end{aligned}}$$

Momentum space (will not be discussed in this talk)

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{k},\boldsymbol{K})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\delta^3(\boldsymbol{k}_i+\boldsymbol{k}_j-\boldsymbol{K})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle$$

$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{k})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle$$

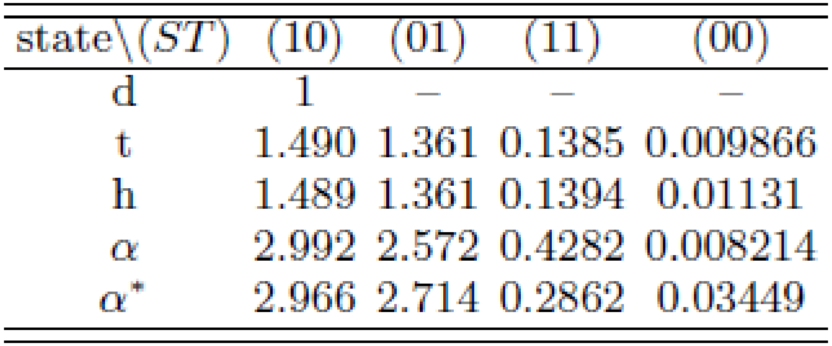
Potential plot (Argonne V8’)



Most attractive

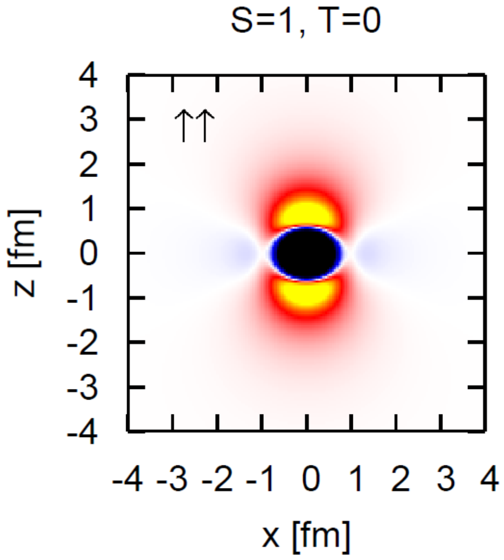
L=1

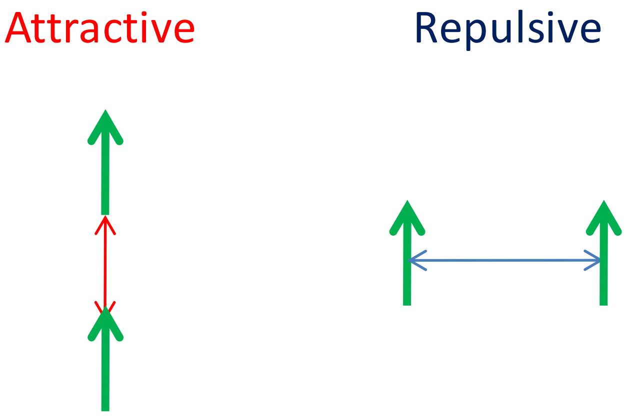
Pair numbers in ST channel



state\(ST)	(10)	(01)	(11)	(00)
d	1	–	–	–
t	1.490	1.361	0.1385	0.009866
h	1.489	1.361	0.1394	0.01131
α	2.992	2.572	0.4282	0.008214
α^*	2.966	2.714	0.2862	0.03449

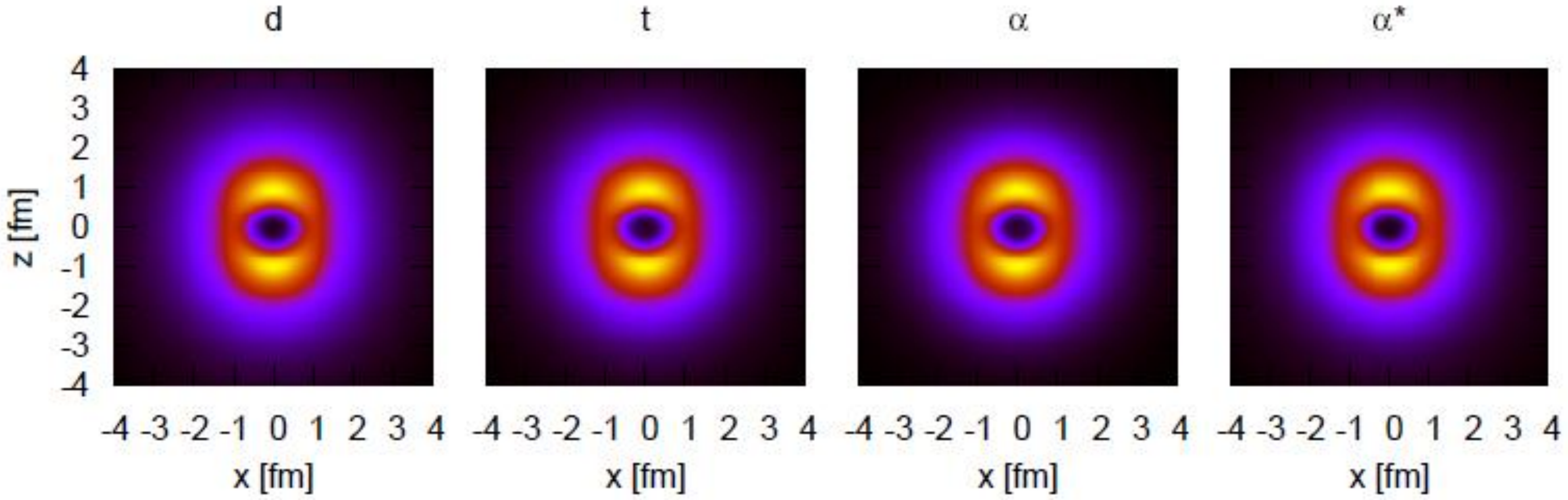
Two-body density (SM_S=11, TM_T=00)


S=1, T=0


Attractive

Repulsive

$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

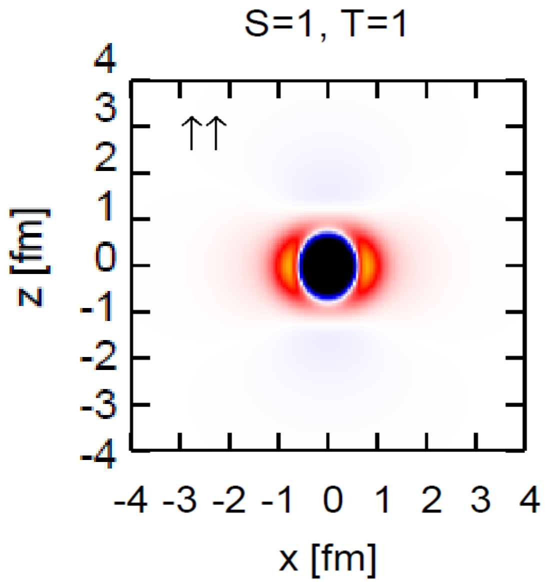

d

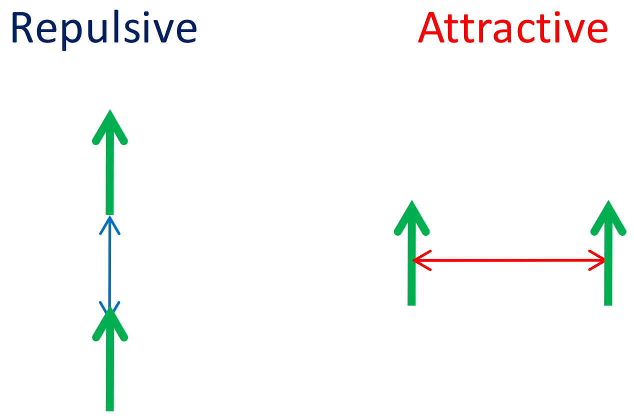
t

α

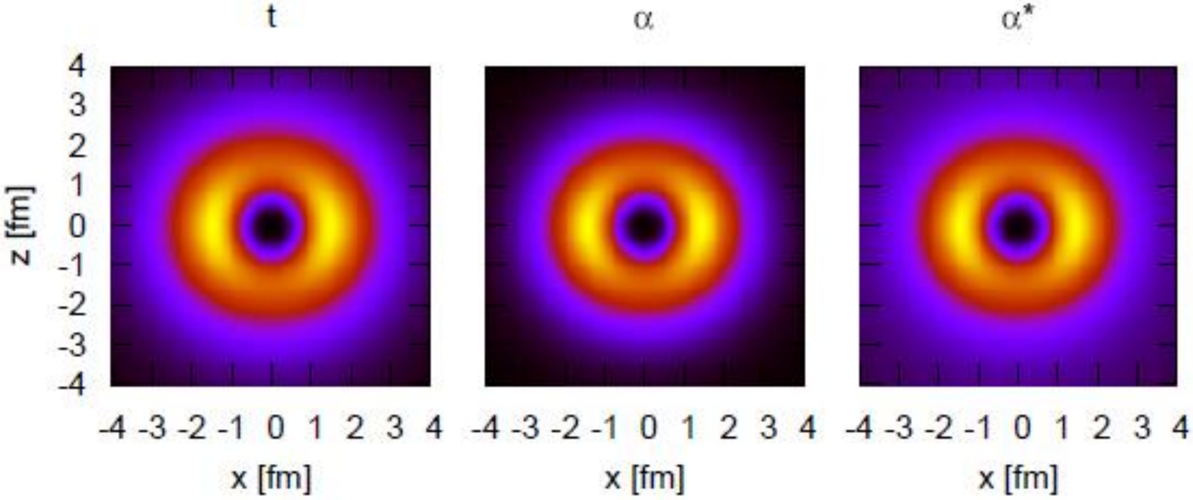
α*

Two-body density (SM_S=11, TM_T=11)

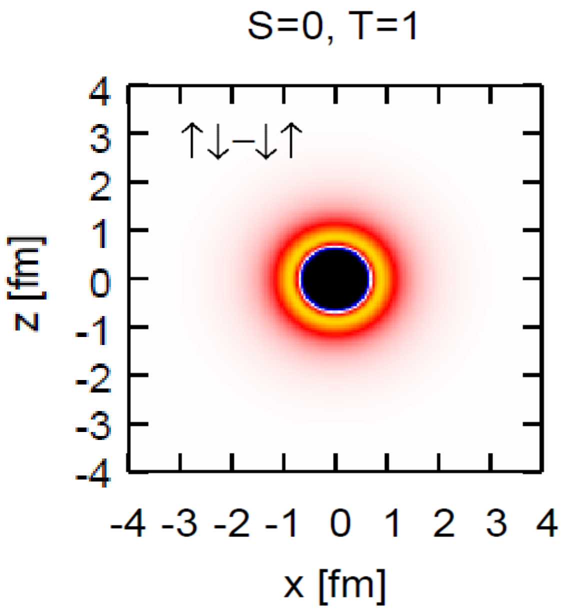


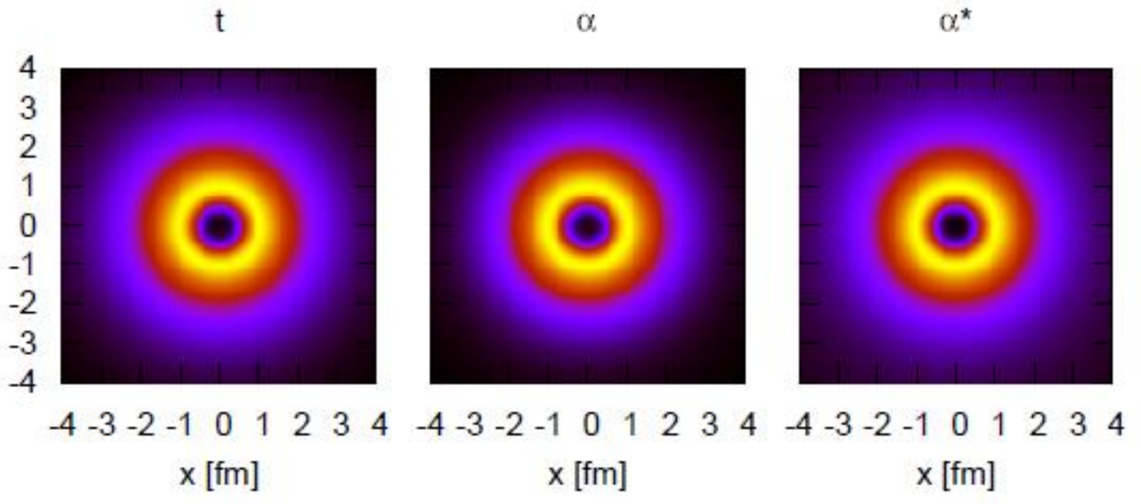


$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$



Two-body density (SM$_S$=00, TM$_T$=11)





SM$_S$=00, TM$_T$=11 channel -> small components in w. f.

One-to-one correspondence to the potential for all ST channels

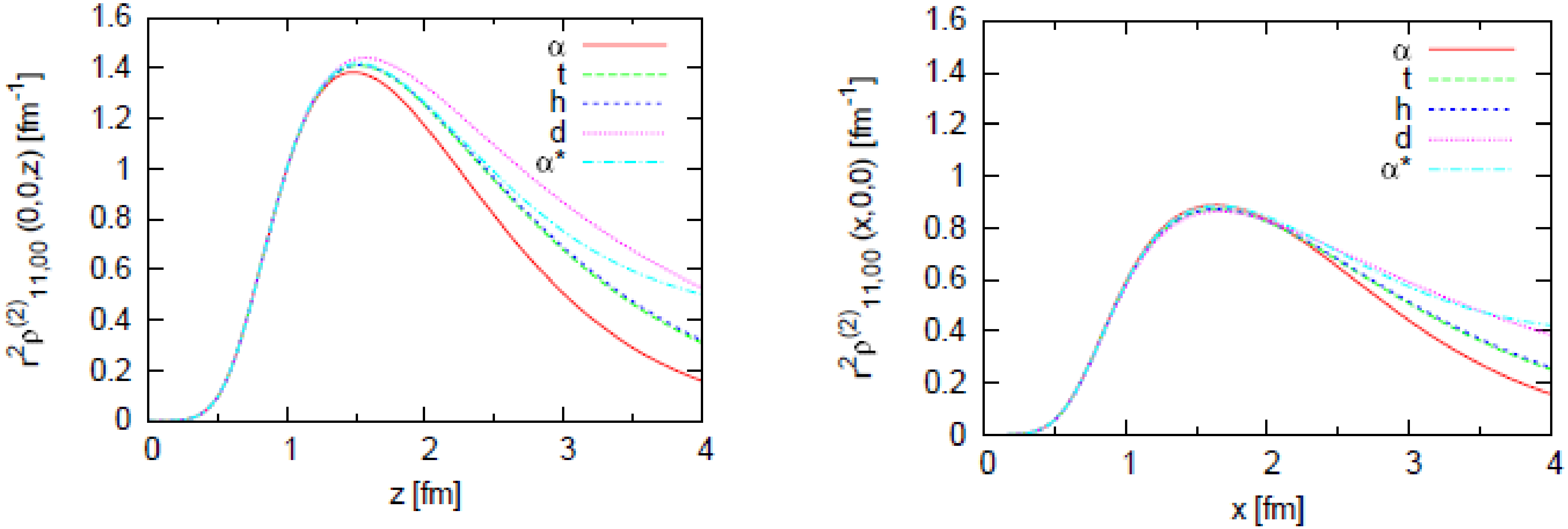
Relevance to density functional theory

Y. Suzuki, W. H., Nucl. Phys. A 818, 188 (2009).

Universality at short distances
(SM_S=11, TM_T=00)

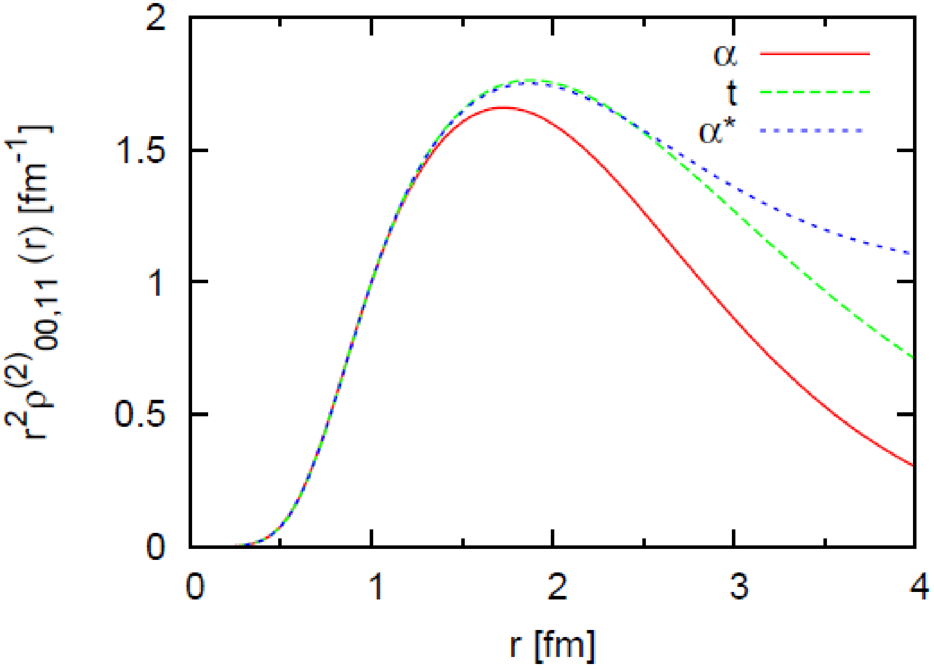
Density cut along z- and x-direction

Normalized at 1 fm of z axis



Universal behavior at short distances of central and tensor correlations

Universality at short-distances (SM_S=00, TM_T=11)



Scaling at short distance is possible

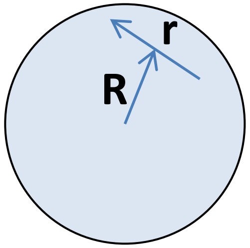
Summary

- Highly correlated many-body system (d, t, h, α, α*)
  - Correlated Gaussian with global vectors
  - Stochastic variational method
- One-body densities
  - Quite different
- Two-body densities
  - One-to-one correspondence to two-body potential
  - Universality at short distances

H. Feldmeier, W.H., T. Neff, Y. Suzuki, in preparation.

- Outlook
  - Two-body density with two variables

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle$$



  - More particle systems (A>4)