RG analysis of coupled-channel scattering and the system ^{7}Li(p,n)^{7}Be

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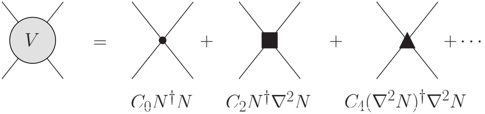
Outline

- Renormalization Group approach to pionless nuclear EFT
- Coupled-channel nuclear scattering analyzed with RG
- ^{7}Li(p,n)^{7}Be system: a physics case

Pionless Nuclear EFT

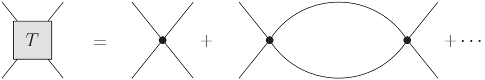
- high momentum scale $\simeq m_\pi$: $Q \lesssim m_\pi$, $E \lesssim 20$ MeV for NN
- contact interactions (with derivatives) $\Longrightarrow$ delta-functions



Weinberg (1990), Kaplan, Savage, Wise (1998), Kong, Ravndal (1999), ...
Beane, Bertulani, Cohen, Hammer, Higa, Gelman, van Kolck, Phillips, Rupak, ...

reviews — Bedaque, van Kolck (2002), Epelbaum (2006)

- loops divergent (couple to arbitrary high momenta)



- need to regularize and renormalize

$\Longrightarrow$ gives rise to RG treatment of nuclear $\not\pi$EFT

Birse, McGovern, Richardson (1999); Birse, Barford (2003); Birse, Ando (2008)

RG Approach

- introduce a cutoff scale μ: $0 < \mu < M_{\text{hep}}$
- all physics on scales larger than μ is integrated out
- separation of scales is important for this to work

- V depends on μ to compensate μ-dependence of loops

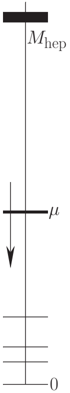
$\Longrightarrow$ RG equation governs the running of V as we change μ

- as we lower μ, more and more physics gets integrated out
- finally, no scales, other than μ, are left

$\Longrightarrow$ Scale-free system (fixed point of RG equation):

  - trivial: no interaction (zero scattering length)
  - nontrivial: infinite scattering length

$\Longrightarrow$ physical systems as perturbations around fixed points



RG:

- shows what counting to use: depends on the starting fixed point
- takes care of the renormalization

RG for Nuclear $\not\pi$EFT

Lippmann-Schwinger Equation ($p = \sqrt{2mE}$):

$$T(p,q,q') = V(p,q,q',\mu) + \int \frac{d^3q''}{(2\pi)^3} V(p,q,q'',\mu) G(p,q'') T(p,q'',q'');$$

assume V contains only energy-dependent delta-functions; then

$$T(p) = V(p,\mu) + V(p,\mu) J(p,\mu) T(p),$$

where

$$J(p,\mu) = 2m \int \frac{d^3q}{(2\pi)^3} \frac{1}{p^2 - q^2 + \mathrm{i}0} = -\frac{m}{2\pi}(\mu + \mathrm{i}\, p);$$

momentum-dependent terms in V are just technically complicated, but are not relevant for counting and for on-shell T matrix

T should not depend on $\mu \Longrightarrow$ RG Equation

RG Equation

differentiate the LS Equation (and use it to get rid of T):

$$\frac{\partial V}{\partial \mu} = -V \frac{\partial J}{\partial \mu} V;$$

now express all quantities in terms of μ:

$$\hat{p} = p/\mu, \quad \hat{\delta} = \delta/\mu, \quad \hat{V} = \frac{\mu}{2\pi} m V;$$

finally,

$$\mu \frac{\partial \hat{V}}{\partial \mu} = \hat{p} \frac{\partial \hat{V}}{\partial \hat{p}} + \hat{V} + \hat{V}^2.$$

- trivial fixed point: $\hat{V} = 0$
- nontrivial fixed point: $\hat{V} = -1$
- linearize RG equation and do perturbations around fixed points
- perturbations should be analytic in $p^2 \propto E$

Counting and Expansion

perturbations around fixed points

- trivial fixed point: no fine-tuning (natural scattering length)

$$\hat{V}(\hat{p},\mu) = \sum_n C_n \, \mu^\nu \, \hat{p}^{2n}, \quad \nu = 2n+1$$

  perturbative: $f = -a + \mathrm{i}pa + \cdots$

- nontrivial fixed point: fine-tuning (large scattering length)

$$\hat{V}(\hat{p},\mu) = -1 + \sum_n C_n \, \mu^\nu \, \hat{p}^{2n}, \quad \nu = 2n-1$$

  enhancement leads to resummation: $f = (-1/a - \mathrm{i}p + \cdots)^{-1}$

straightforward to include — do not change the RG scaling/counting:

- momentum-dependent interactions
  Birse, McGovern, Richardson (1998)
- Coulomb interaction
  Ando, Birse (2008)

RG for Coupled Channels

still with only energy-dependent interactions:

$$\mathbf{T}(p,\delta) = \mathbf{V}(p,\delta,\mu) + \mathbf{V}(p,\delta,\mu)\mathbf{J}(p,\delta,\mu)\mathbf{T}(p,\delta),$$

the Green's function loop integral is

$$\mathbf{J}(p,\delta,\mu) = -\begin{pmatrix} \frac{m_1}{2\pi}(\mu + \mathrm{i}p_1) & 0 \\ 0 & \frac{m_2}{2\pi}(\mu + \mathrm{i}p_2) \end{pmatrix},$$

the second channel threshold energy Δ is assumed to be such that $\delta = \sqrt{2m_1\Delta} \sim p$ is small (another small scale);

$$p_1 \equiv p = \sqrt{2m_1 E}, \quad p_2 \equiv p' = \sqrt{\frac{m_2}{m_1}(p^2 - \delta^2)}, \quad \mathbf{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix};$$

$\Longrightarrow$ RG equation for the rescaled potential,

$$\hat{\mathbf{V}} = \frac{\mu}{2\pi}\mathbf{M}^{1/2}\mathbf{V}\mathbf{M}^{1/2},$$

RG Equation

$$\mu \frac{\partial \hat{\mathbf{V}}}{\partial \mu} = \hat{p} \frac{\partial \hat{\mathbf{V}}}{\partial \hat{p}} + \hat{\delta} \frac{\partial \hat{\mathbf{V}}}{\partial \hat{\delta}} + \hat{\mathbf{V}} + \hat{\mathbf{V}}^2;$$

we can rewrite it as a linear equation for $\hat{\mathbf{V}}^{-1}$:

$$\mu \frac{\partial}{\partial \mu} \hat{\mathbf{V}}^{-1} = \hat{p} \frac{\partial}{\partial \hat{p}} \hat{\mathbf{V}}^{-1} + \hat{\delta} \frac{\partial}{\partial \hat{\delta}} \hat{\mathbf{V}}^{-1} - \hat{\mathbf{V}}^{-1} - \mathbf{1}$$

this equation can be solved exactly (analyticity constraint applies):

$$\hat{\mathbf{V}}(\hat{p}, \hat{\delta}, \mu)^{-1} = -\mathbf{1} - \sum_{n,m} \mathbf{C}_{nm} \, \mu^{2m+2n-1} \, \hat{p}^{2n} \, \hat{\delta}^{2m}$$

let c_1 and c_2 be eigenvalues of $\mathbf{C}_{00}$

- they have the meaning of inverse scattering lengths in the "diagonal" channels
- the counting is set by the sizes of c_1 and c_2

Counting for Coupled Channels

$$\hat{\mathbf{V}}(\hat{p},\hat{\delta},\mu)^{-1} = -\mathbf{1} - \sum_{n,m} \mathbf{C}_{nm}\,\mu^{2m+2n-1}\,\hat{p}^{2n}\,\hat{\delta}^{2m}$$

there are three possibilities:

a: both c_1 and c_2 are natural, $c_{1,2} \gg \mu$

$$\hat{\mathbf{V}}(\hat{p},\hat{\delta},\mu) = -\mathbf{C}_{00}^{-1}\mu + \mathbf{C}_{00}^{-1}{\sum_{n,m}}' \mathbf{C}_{nm}\,\mu^{2m+2n+1}\,\hat{p}^{2n}\,\hat{\delta}^{2m}\,\mathbf{C}_{00}^{-1} + \cdots$$

$\Longrightarrow$ perturbative in both channels

b: both c_1 and c_2 are unnaturally small, $c_{1,2} \ll \mu$

$$\hat{\mathbf{V}}(\hat{p},\hat{\delta},\mu) = -\mathbf{1} + \sum_{n,m} \mathbf{C}_{nm}\,\mu^{2m+2n-1}\,\hat{p}^{2n}\,\hat{\delta}^{2m} + \cdots$$

$\Longrightarrow$ enhancement μ^{-2} in both channels, non-perturbative

Cohen, Gelman, van Kolck (2004)

c: ... ?

Counting for Single State at Threshold

c: c_1 is unnaturally small, c_2 is natural, $c_1 \ll \mu \ll c_2$
$\Longrightarrow$ "intermediate" (also in the amount of fine-tuning!)

- disentangle the two diagonal channels:

$$\mathbf{u} = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \to c_1, \quad \mathbf{v} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix} \to c_2 : \quad \mathbf{C}_{00} = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

$$\mathbf{P}_1 = \mathbf{u}\,\mathbf{u}^{\mathrm{T}}, \quad \mathbf{P}_2 = \mathbf{v}\,\mathbf{v}^{\mathrm{T}}, \quad \mathbf{P}_3 = \mathbf{u}\,\mathbf{v}^{\mathrm{T}}, \quad \mathbf{P}_4 = \mathbf{v}\,\mathbf{u}^{\mathrm{T}};$$

- rewrite the potential in the new basis:

$$\hat{\mathbf{V}}(\hat{p}, \hat{\delta}, \mu)^{-1} = -\mathbf{P}_1 - \mathbf{P}_2 - \sum_i \mathbf{P}_i \sum_{n,m} c_{nm}^{(i)}\, \mu^{2m+2n-1}\, \hat{p}^{2n}\, \hat{\delta}^{2m}$$

- the resulting (after doing some algebra) counting:
  - "strong" (**u**) channel, $\mathbf{P}_1$: enhancement by μ^{-2}
  - mixing channel, $\mathbf{P}_{3,4}$: enhancement by μ^{-1}
  - "weak" (**v**) channel, $\mathbf{P}_2$: no enhancement

RG: Results

- three different countings, depending on eigenvalues of $\mathbf{C}_{00}$
- the expansion is for the exact solution of RG equation $\Longrightarrow$ renormalization is taken care of
- for the counting with single state at threshold, disentangling non-perturbative and perturbative channels is crucial
- Coulomb can be included (by dressing the Green's function), does not change the counting

$$\hat{\mathbf{V}}(\hat{p},\hat{\delta},\mu)^{-1} = -\mathbf{1} - \sum_{n,m} \mathbf{C}_{nm}\,\mu^{2m+2n-1}\,\hat{p}^{2n}\,\hat{\delta}^{2m} \qquad \Longrightarrow$$

$$\hat{\mathbf{V}}(\hat{p},\hat{\delta},\hat{\kappa},\mu)^{-1} = -\mathbf{1} + \hat{\boldsymbol{\kappa}}\,2\ln\frac{\mu}{\lambda} - \sum_{n,m,l} \mathbf{C}_{nml}\,\mu^{2m+2n+l-1}\,\hat{p}^{2n}\,\hat{\delta}^{2m}\hat{\kappa}^{l}$$

now, time to try and see if it works for a real system!

^{7}Li(p,n)^{7}Be Coupled Channels

why interesting?

- structure of excited states of ^{8}Be
- nucleosynthesis: abundance of ^{7}Li

- also, ^{7}Li(p,n)^{7}Be is a neutron source for neutron capture therapy

- ^{7}Li, ^{7}Be: $J^P = 3/2^-$
- $E = 0$ corresponds to ^{7}Li p at rest $\longrightarrow$ channel 1
- ^{7}Be n threshold is at $E = \Delta = 1.6442$ MeV (c.m.) $\longrightarrow$ channel 2
- neutron threshold momentum $\delta = \sqrt{2m_1\Delta} = 51.95$ MeV
- Coulomb interaction in ^{7}Li p channel: inverse Bohr radius $\kappa_1 \equiv \kappa = 3\alpha m_1 = 17.96$ MeV$\sim \delta$

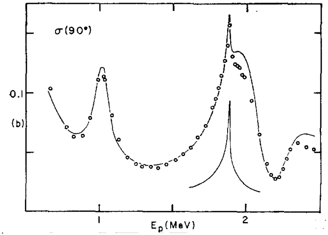
^{7}Li(p,n)^{7}Be Features

- cusp at neutron threshold in ^{7}Li(p,p)^{7}Li (elastic) cross section
- LARGE ^{7}Be(n,p)^{7}Li thermal cross section, $\sigma_{t^\circ} = 38400$ b
- both effects due to $^5S_2$ partial wave
- ^{8}Be 2^- state is very close to neutron threshold
- also, no features in $^5S_2$ partial wave below neutron threshold

— a good candidate to be studied
employing counting for single near-threshold state

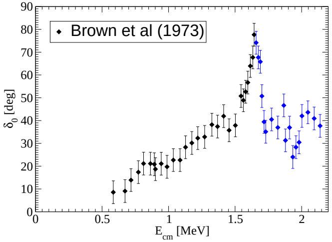
other pw's are important
(most notably, $^5P_1$ and $^5P_3$)

$\Longrightarrow$ pw analysis is needed
$\longrightarrow$ Brown et al (1973)

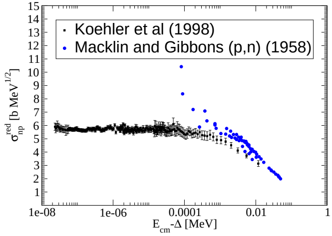


Data

^{7}Li(p,p)^{7}Li



^{7}Be(n,p)^{7}Li



phase analysis: $^5S_2$ phase shift δ_0

data: $\sigma^{\rm red}_{\rm np} = \sigma_{\rm np}\sqrt{E-\Delta}$
only $^5S_2$ at these energies

- M&G data (measured in ^{7}Li(p,n)^{7}Be) systematically higher than data by Koehler $\Longrightarrow$ discard M&G data
- however, M&G data used in Brown (inelasticities above neutron threshold) $\Longrightarrow$ discard also δ_0 above n threshold
- below neutron threshold inelasticities are small $\Longrightarrow$ no reason to doubt δ_0 there

LS equation

now is

$$\mathbf{T}_C(p,\delta,\kappa) = \mathbf{V}(p,\delta,\kappa) + \mathbf{V}(p,\delta,\kappa)\mathbf{J}^R(p,\delta,\kappa)\mathbf{T}_C(p,\delta,\kappa),$$

where

$$\mathbf{J}^R(p,\delta,\kappa) = \begin{pmatrix} \frac{m_1}{2\pi} j_1^R & 0 \\ 0 & \frac{m_2}{2\pi} j_2^R \end{pmatrix},$$

with

$$j_1^R(p,\delta,\kappa) = -\kappa\left(2h(\eta) + \mathrm{i}\frac{C_\eta^2}{\eta}\right), \quad j_2^R(p,\delta,\kappa) = -\mathrm{i}p'\,.$$

$\eta = \kappa/p$, $h(\eta) = \mathrm{Re}\,\psi(\mathrm{i}\eta) - \ln\eta$, $C_\eta^2 = 2\pi\eta/(\exp 2\pi\eta - 1)$;

divergent terms in j_i^R are cancelled by terms in the potential
the remainders both of the potential and of the loop integrals are finite
from now on

LO

$$\mathbf{V} = 2\pi a_1\, \mathbf{M}^{-1/2}\mathbf{P}_1\mathbf{M}^{-1/2}$$

a_1 scattering length in strong channel; mixing angle ϕ enters via $\mathbf{P}_1$

$$\mathbf{T}^{\rm LO} = -2\pi\, \left[-\frac{1}{a_1} + \cos^2\phi\, j_1^R + \sin^2\phi\, j_2^R\right]^{-1} \mathbf{M}^{-1/2}\mathbf{P}_1\mathbf{M}^{-1/2}.$$

relation to observables:

$$\begin{aligned} \frac{e^{2\mathrm{i}\delta_0}-1}{2\mathrm{i}p} &= -\frac{m_1}{2\pi}\, T_{11}\, C_\eta^2, \\ \sigma_{\rm np} &= \frac{4\pi(2J+1)}{(2s_1+1)(2s_2+1)}\, \frac{p}{p'}\, \frac{m_1 m_2}{4\pi^2}\, \left|-C_\eta\, T_{21}\right|^2, \end{aligned}$$

use at threshold

$$\delta_0 = 83.3^\circ, \quad \sigma_{\rm np}^{\rm red} = 5.75 \text{ b MeV}^{1/2}$$

- $a_1 = -0.090\ \text{MeV}^{-1} = -17.6$ fm: large scattering length
- $\phi = 0.814 = 46.6^\circ$: $\sim 50/50$ mixing of physical channels

NLO and NNLO

at NLO,

$$\mathbf{V}^{-1} = -\frac{1}{2\pi}\mathbf{M}^{1/2}\left[\left(-\frac{1}{a_1}+\frac{r_0}{2}p'^2\right)\mathbf{P}_1 - \frac{1}{a_2}\mathbf{P}_2\right]\mathbf{M}^{1/2};$$

in the basis of $\mathbf{C}_{00}$ eigenstates $\Longrightarrow$ NLO mixing parameter $= 0$

$$\mathbf{T}^{\mathrm{NLO}} = -2\pi\,\mathbf{M}^{-1/2}\left[\frac{\mathbf{P}_1}{f_1^{-1} - f_2 f_{12}^{-2}} + \frac{f_2\mathbf{P}_2}{1 - f_1 f_2 f_{12}^{-2}} - \frac{f_{12}^{-1} f_2(\mathbf{P}_3 + \mathbf{P}_4)}{f_1^{-1} - f_2 f_{12}^{-2}}\right]\mathbf{M}^{-1/2},$$

$$\begin{aligned} f_1^{-1} &= -\frac{1}{a_1} + \frac{r_0}{2}p'^2 - j_1^R\cos^2\phi - j_2^R\sin^2\phi, \\ f_2^{-1} &= -\frac{1}{a_2} - j_1^R\sin^2\phi - j_2^R\cos^2\phi, \\ f_{12}^{-1} &= \left(j_1^R - j_2^R\right)\cos\phi\sin\phi. \end{aligned}$$

we can expand in accordance with the counting

NLO and NNLO

expanded **T** at NLO:

$$\mathbf{T}_{\rm exp}^{\rm NLO} = -2\pi\,\mathbf{M}^{-1/2}$$

$$\times\left[\frac{\mathbf{P}_1}{-\frac{1}{a_1}+\frac{r_0}{2}p'^2 - j_1^R\cos^2\phi - j_2^R\sin^2\phi + a_2\left(j_1^R - j_2^R\right)^2\cos^2\phi\sin^2\phi}\right.$$

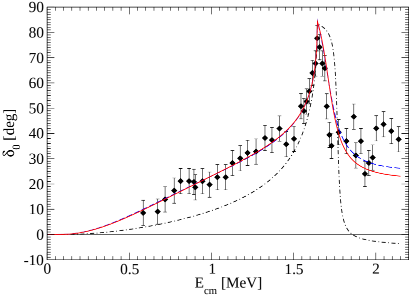
$$\left. - a_2\mathbf{P}_2 + \frac{a_2\left(j_1^R - j_2^R\right)\cos\phi\sin\phi(\mathbf{P}_3+\mathbf{P}_4)}{-\frac{1}{a_1} - j_1^R\cos^2\phi - j_2^R\sin^2\phi}\right]\mathbf{M}^{-1/2};$$

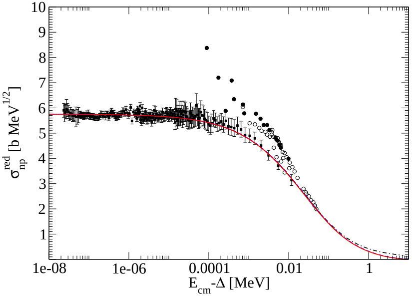
at NNLO: + effective range (the leading term) in the mixing channel:

$$\delta\mathbf{V}^{-1}_{\rm NNLO} = -2\pi\,\mathbf{M}^{-1/2}\left[\frac{r_1}{2}p'^2\,\mathbf{P}_3 + \frac{r_1^*}{2}p'^2\,\mathbf{P}_4\right]\mathbf{M}^{-1/2};$$

T is calculated and expanded in the same way as NLO
(very lengthy, but straightforward, expressions are omitted here)

Results: Observables and Parameters





Order	a_1 [MeV^{-1}]	ϕ	a_2 [MeV^{-1}]	r_0 [MeV^{-1}]	r_1 [MeV^{-1}]
LO	-0.090	0.814	–	–	–
NLO	-0.094	0.845	-0.003	0.023	–
NNLO	-0.094	0.853	-0.003	0.021	-0.003

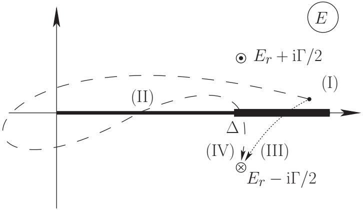
^{8}Be Pole

position: $E = E_r - \mathrm{i}\Gamma/2 = 1.70 - \mathrm{i}0.07$ MeV on sheet IV
Koehler et al (1998): $E = 1.64 - \mathrm{i}0.06$ MeV on sheet IV
— $\sim$ consistent, but ours is significantly farther away from threshold

Riemann sheets:

Frazer, Hendry (1964)

- $\mathrm{Im}\, p_1 > 0,\ \mathrm{Im}\, p_2 > 0$: I (physical);
- $\mathrm{Im}\, p_1 < 0,\ \mathrm{Im}\, p_2 > 0$: II;
- $\mathrm{Im}\, p_1 < 0,\ \mathrm{Im}\, p_2 < 0$: III;
- $\mathrm{Im}\, p_1 > 0,\ \mathrm{Im}\, p_2 > 0$: IV



Breit-Wigner resonance: a pole on sheet III close to physical region
unitarity: $\mathbf{A} = \mathrm{Res}\, \mathbf{S}|_E$ Hermitian rank-1 matrix; $\sum_i |A_{ii}| \equiv \sum_i \Gamma_i = \Gamma$

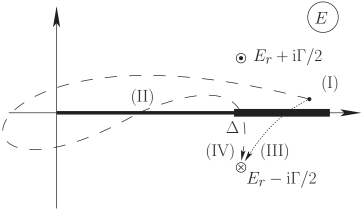
a pole on sheet IV, far from physical region, breaks these conditions:
A is not Hermitian;
$\sum_i |A_{ii}| \equiv \sum_i \Gamma_i \neq \Gamma$: $\Gamma_p = 2.41$ MeV, $\Gamma_n = 0.34$ MeV, $\sum_i \Gamma_i/\Gamma = 19.8$

— a *shadow* resonance

Eden, Taylor (1964); Hale, Brown, Jarmie (1987)

^{8}Be Pole and Zeros

- pole far from physical region
- how can it affect observables?



conjugated zeros of S_{22}:

Eden, Taylor (1964)

$S_{22} = 0$ at $E = E_r \pm i\Gamma/2 = 1.70 \pm i0.07$ on sheet I (physical)

$\Longrightarrow |S_{22}|$ small at $E \sim E_r$ in physical region
$\Longrightarrow |S_{11}|$ small, $|S_{12}| \sim 1$ at $E \sim E_r$ in physical region above neutron threshold (via unitarity)

— large inelasticity in the diagonal channel, ^{7}Li(p,p)^{7}Li, right above neutron threshold
— close to unitarity limit in ^{7}Be(n,p)^{7}Li (and crossed) channel

— seen on data

Summary

- RG allows to simplify analysis of $\not{\pi}$EFT
- RG for two coupled channels in S-wave:
  - three types of countings, depending on sizes of two parameters c_1 and c_2:
    - perturbative
    - non-perturbative in both channels
    - intermediate
  - results in an effective range expansion-like calculation, using one of the countings
- ^{7}Li(p,n)^{7}Be is a very good example of a system that obeys the intermediate counting
  - large mismatch between scattering lengths in the two diagonal channels
  - large mixing between the physical channels
  - data are described well
- other systems that might be similar to ^{7}Li(p,n)^{7}Be?