Properties of alpha- and nucleon-clusters in the light of EFT

Renato Higa

Kernfysisch Versneller Instituut
University of Groningen

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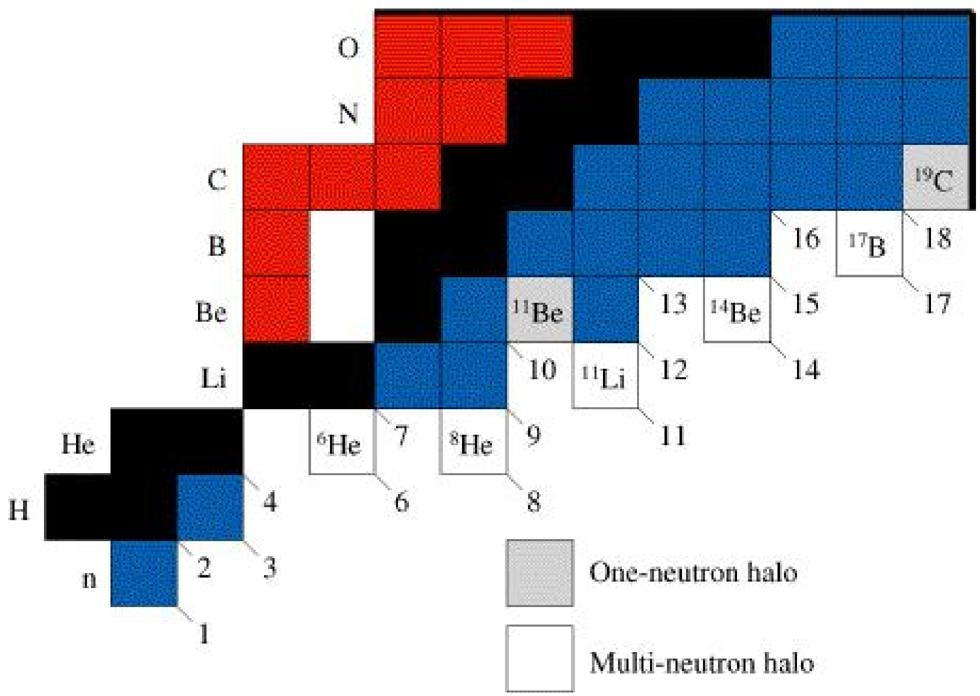
Properties of alpha- and nucleon-clusters in the light of EFT

Outline

- motivation
- nuclear halo/clusters
  - ⋆ the $\alpha\alpha$ interaction
  - ⋆ $p\alpha$ system
- Summary and outlook

Halo nuclei: separation of scales



- excitation of each cluster $\sqrt{m_c E_c^*} \sim M_{hi}$
- binding among clusters $\sqrt{m_c E_B} \sim M_{lo}$
- desired feature for EFT
- expansion in powers of M_{lo}/M_{hi}

⋆ ^{11}Be , ^{19}C, ^{11}Li , ^{6}He , ^{14}Be, ^{8}He, ^{8}B, ^{17}Ne, ...

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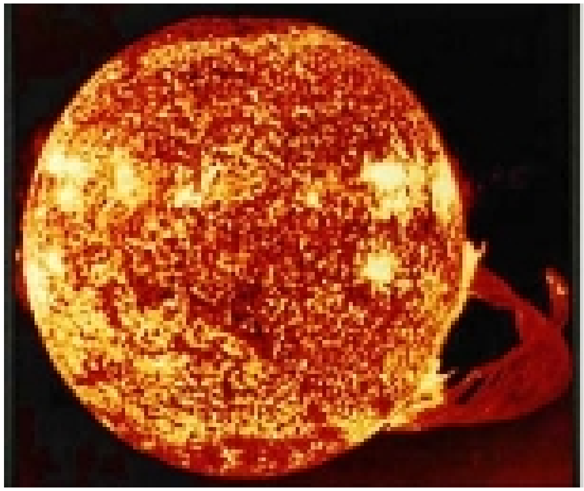
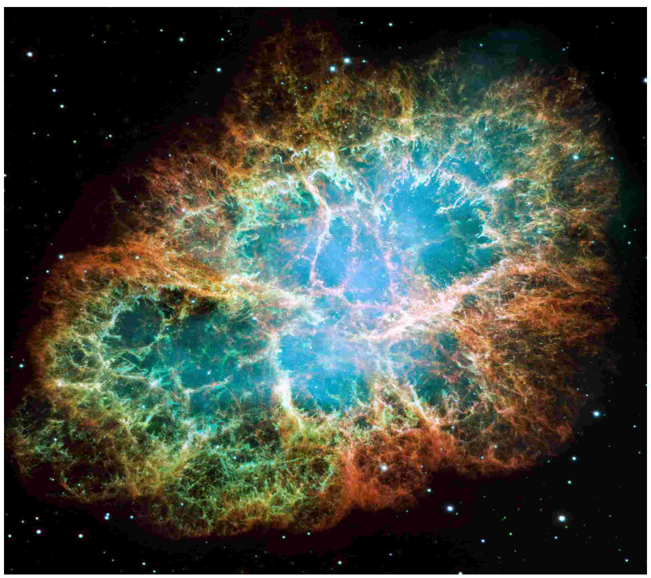


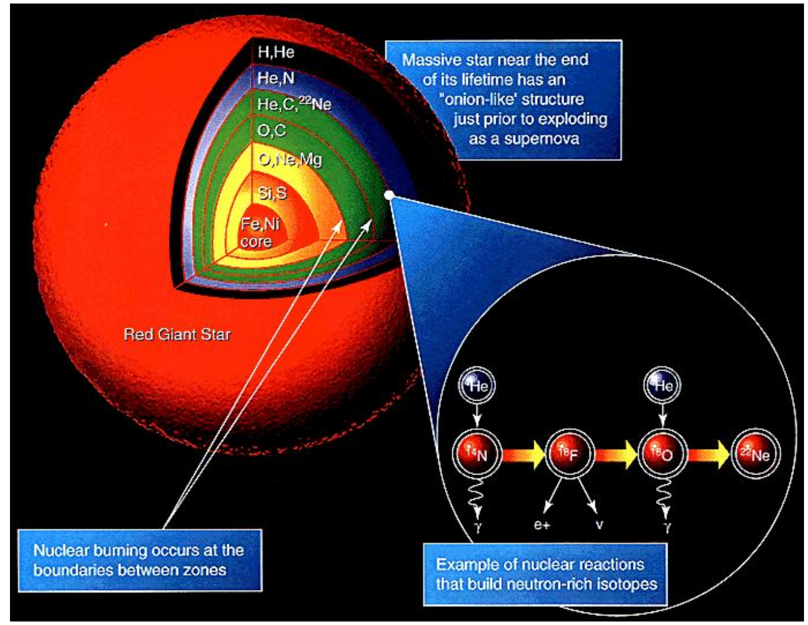
$\mathrm{p} + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$

solar neutrinos



${}^{9}\mathrm{Be}(\frac{1}{2}_{+}) + \alpha \rightarrow {}^{12}\mathrm{C} + \mathrm{n}$

core-colapse supernovae

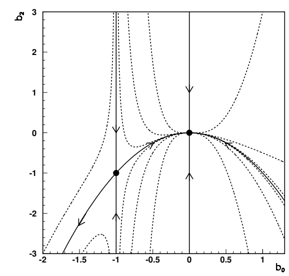


${}^{8}\mathrm{Be} + \alpha \rightarrow {}^{12}\mathrm{C} + \gamma$

red giants

EFT and universality

- two-body systems
  - ⋆ RG equations ⇒ non-trivial fixed point
  - ⋆ non-relativistic conformal invariance
  - ⋆ similar to critical phenomena
  - ⋆ scattering length as parameter control



- three-body systems
  - ⋆ RG-flow tends twds a lymit cycle instead of a fixed point (Wilson, 70's)
  - ⋆ Efimov states: $E^{(n+1)} \sim 515 \times E^{(n)}$

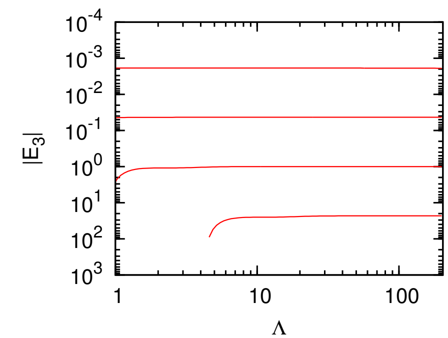
⋆ Fedorov, Jensen, Garrido, ...

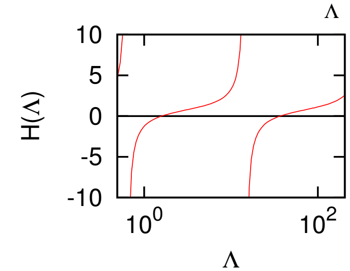
⋆ Amorim, Frederico, Tomio, Delfino, Adhikari, ...

⋆ Greene, d'Incao, von Stecher, ...

⋆ Bedaque, Hammer, van Kolck, Braaten, Platter, ...

⋆ ...

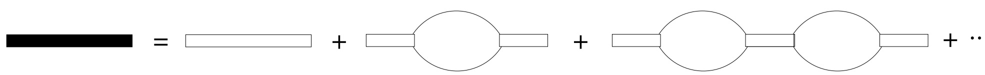




(Hammer, RH, EPJA 37,193)

halo/cluster EFT for $\alpha\alpha$ system

$$\mathcal{L} = \phi^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{4\mu} \right] \phi + \sigma\, d^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{8\mu} - \Delta \right] d + g \left[d^\dagger \phi\phi + (\phi\phi)^\dagger d \right] + \cdots\,,$$



$$\Delta \sim M_{lo} \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{-\Delta + i\epsilon} \sim \frac{1}{M_{lo}} \qquad (NN)$$

$$\Delta \sim M_{lo}^2/\mu \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{q_0 - \boldsymbol{q}^2/8\mu - \Delta + i\epsilon} \sim \frac{\mu}{M_{lo}^2} \qquad (\alpha\alpha)$$

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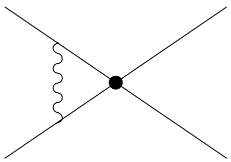
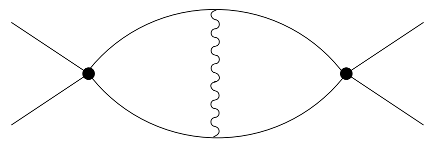
resummation of Coulomb photons required

$$k_C = Z_\alpha^2 \alpha_{em} m_\alpha / 2 \sim M_{hi}$$

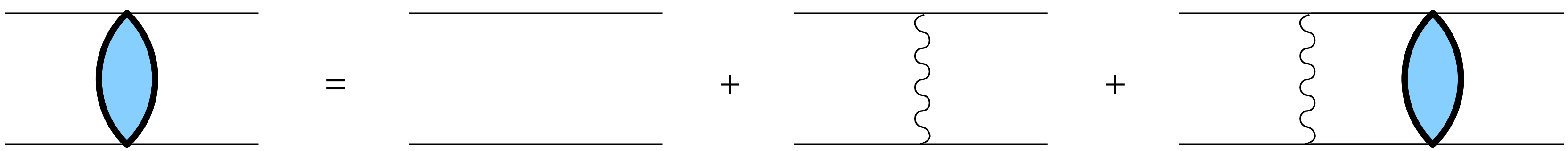


$$\sim \frac{k_C}{k} \equiv \eta \,,$$



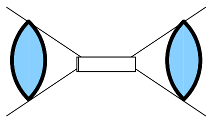
$$\sim k_C m \left[\frac{1}{2} \ln \frac{\mu}{2k} + \cdots \right]$$

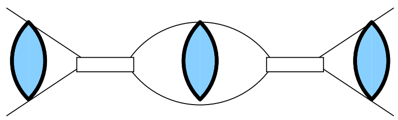
non-perturbative Coulomb (Kong and Ravndal, NPA 665, 137)

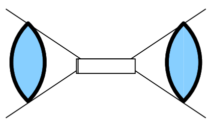
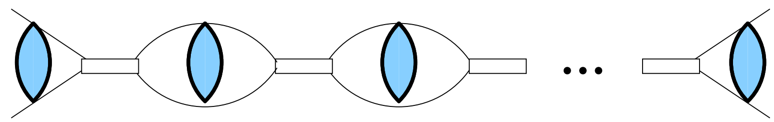


$$G_C^{(\pm)}(E) = m_\alpha \int \frac{d^3 q}{(2\pi)^3} \frac{|\chi_q^{(\pm)}\rangle\langle\chi_q^{(\pm)}|}{m_\alpha E - \boldsymbol{q}^2 \pm i\epsilon}$$

$$T_{CS} = \langle \chi_{k'}^{-} | \, \hat{V}_S \, | \chi_k^{+} \rangle + \langle \chi_{k'}^{-} | \, \hat{V}_S \, G_C^{+} \, \hat{V}_S \, | \chi_k^{+} \rangle + \cdots$$

$T_{CS}^{(0)} =$  $= d(E) \, \chi_{k'}^{(-)\,*}(0) \, \chi_k^{(+)}(0) = d(E) \, C_\eta^{(0)\,2} \, e^{2i\sigma_0} \,,$

$T_{CS}^{(1)} =$  $= d(E) \, C_\eta^{(0)\,2} \, e^{2i\sigma_0} \, d(E) \, J_0(E) \,,$

$T_{CS} =$  $+ \cdots +$ 

$$= C_\eta^{(0)\,2} \, \frac{d(E) \, e^{2i\sigma_0}}{1 - d(E) \, J_0(E)} \,,$$

$$J_0(E) = m_\alpha \int \frac{d^3q}{(2\pi)^3} \, \frac{\chi_q^{(+)}(0) \chi_q^{(+)\,*}(0)}{k^2 - \boldsymbol{q}^2 \pm i\epsilon} = m_\alpha \int \frac{d^3q}{(2\pi)^3} \, \frac{2\pi\eta_q}{e^{2\pi\eta_q} - 1} \, \frac{1}{k^2 - q^2 + i\epsilon}$$

$(\eta_q = 1/a_B q = k_C/q)$

$\alpha\alpha$ scattering

- $0+$ resonance (^{8}Be g.s.):

  $E_R^{\rm LAB} = 184.15 \pm 0.07$ keV , $\Gamma_R^{\rm LAB} = 11.14 \pm 0.50$ eV

  $M_{lo} \approx \sqrt{m_\alpha E_R^{\rm CM}} \sim 20$ MeV, $\quad M_{hi} \sim m_\pi \sim 140$ MeV

- power-counting: $E_{LAB} \leq 3.0$ MeV

- scattering: Afzal *et.al.* (1969)

  ⋆ $E_{LAB} \leq 3.0$ MeV : data from Heydenburg and Temmer (1956)
  ⋆ ERE parameters from Russell *et.al.* (1956), Rasche (1967):
  $a_0 = (-1.65 \pm 0.17) \times 10^3$ fm,
  $r_0 = 1.084 \pm 0.011$ fm $\sim 1/M_{hi}$, $\quad \mathcal{P}_0 = -1.76 \pm 0.22$ fm^3 $\sim 1/M_{hi}^3$

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$$T_{CS} = C_\eta^{(0)\,2}\,\frac{d(E)\,e^{2i\sigma_0}}{1-d(E)\,J_0(E)} = -\frac{4\pi}{m_\alpha}\,\frac{C_\eta^{(0)\,2}\,e^{2i\sigma_0}}{-\frac{1}{a_0}+\frac{r_0}{2}\,k^2-i\epsilon+\frac{4\pi}{m_\alpha}\,J_0(E)}$$

$$= -\frac{4\pi}{m_\alpha}\,\frac{C_\eta^{(0)\,2}\,e^{2i\sigma_0}}{-\frac{1}{a_0^c}+\frac{r_0}{2}\,k^2-\frac{2}{a_B}\,H(\eta)},$$

$$a_B = \frac{1}{k_C} \sim \frac{1}{M_{hi}}$$

$$H(\eta) = \psi(i\eta)+\frac{1}{2i\eta}-\ln(i\eta) \Rightarrow \begin{cases} \overset{\eta\ll 1}{\longrightarrow} \frac{a_B}{2}\,ik \\ \overset{\eta\gg 1}{\longrightarrow} \frac{1}{12}\,(a_B\,k)^2+\frac{1}{120}\,(a_B\,k)^4 \end{cases}$$

- without Coulomb: conformal invariance in ^{8}Be , Efimov spectrum in ^{12}C at LO (RH, Hammer, van Kolck, Nucl. Phys. A 809, 171)
- with Coulomb: ^{8}Be and ^{12}C 0+ states remain close to threshold

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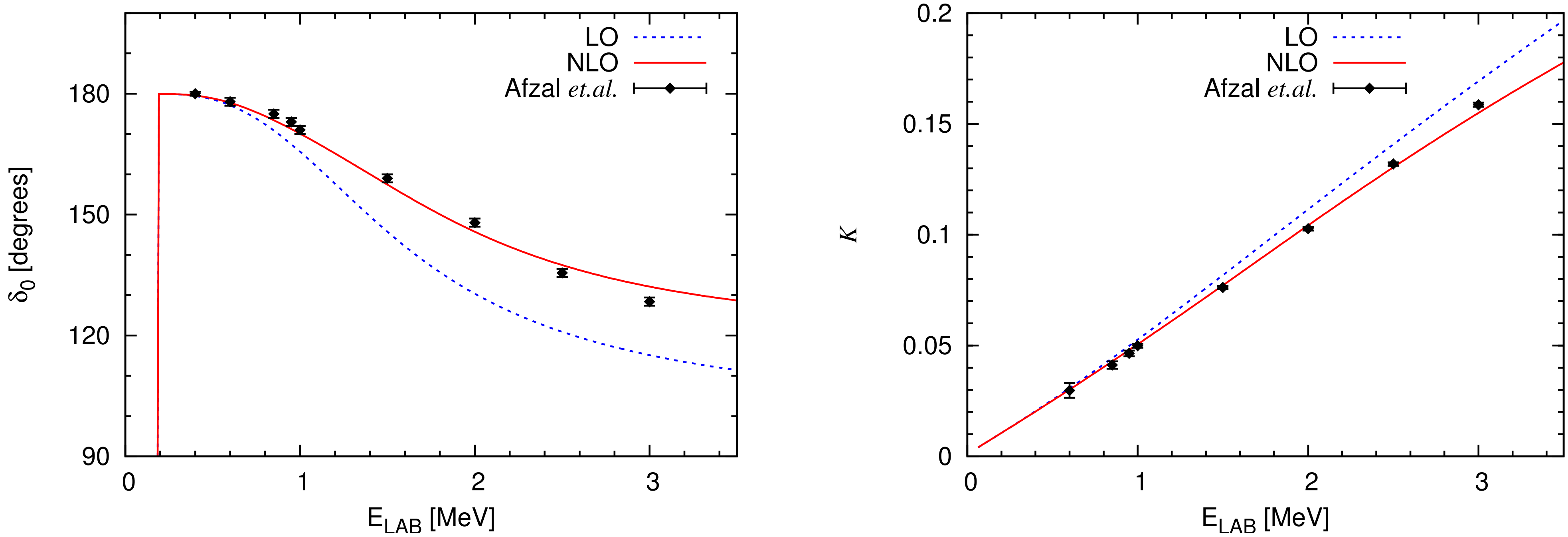
(RH, Hammer, van Kolck, 2008)



	a_0 (10^3 fm)	r_0 (fm)	$\mathcal{P}_0$ (fm^3)
LO	-1.80	1.083	—
NLO	-1.92 ± 0.09	1.098 ± 0.005	-1.46 ± 0.08
Rasche	-1.65 ± 0.17	1.084 ± 0.011	-1.76 ± 0.22

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fine-tuning puzzle

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}^2}{m_\alpha}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{m_\alpha}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{m_\alpha}} \qquad \text{(natural)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}M_{lo}}{m_\alpha}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{m_\alpha}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{m_\alpha}} \qquad \text{(fine-tuned like } NN\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^2}{m_\alpha}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{m_\alpha}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{m_\alpha}} \qquad \text{(fine-tuned to get } E_R\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^3}{M_{hi}m_\alpha}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{m_\alpha}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{m_\alpha}} \qquad \text{(fine-tuned to get } \Gamma_R\text{)}$$

$\sim$ factor of 1000!!!

(Oberhummer *et al.*, Science 289, 88; RH, Hammer, van Kolck, 2008)

$p\alpha$ scattering: $S_{1/2}$, $P_{3/2}$, $P_{1/2}$

$$\mathcal{L}_{\rm LO} = \phi^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_\alpha} \right] \phi + N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N$$

$$+ \eta_{1+}\, t^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2(m_\alpha + m_N)} - \Delta_{1+} \right] t$$

$$+ \frac{g_{1+}}{2} \left\{ t^\dagger \vec{S}^\dagger \cdot \left[N\vec{\nabla}\phi - (\vec{\nabla}N)\phi \right] + \text{H.c.} - r \left[t^\dagger \vec{S}^\dagger \cdot \vec{\nabla}(N\phi) + \text{H.c.} \right] \right\}$$

$$\mathcal{L}_{\rm NLO} = \eta_{0+}\, s^\dagger \left[-\Delta_{0+} \right] s + g_{0+} \left[s^\dagger N\phi + \phi^\dagger N^\dagger s \right] + g'_{1+}\, t^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2(m_\alpha + m_N)} \right]^2 t$$

(Bertulani, Hammer, van Kolck, NPA 712, 37;

Bedaque, Hammer, van Kolck, PLB 569, 159)

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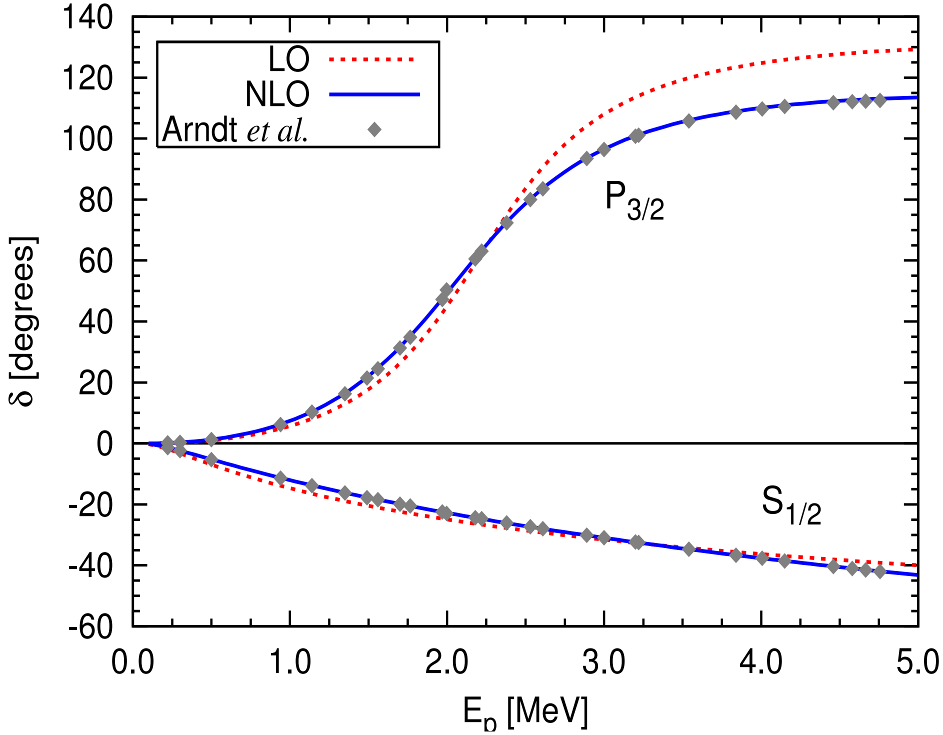
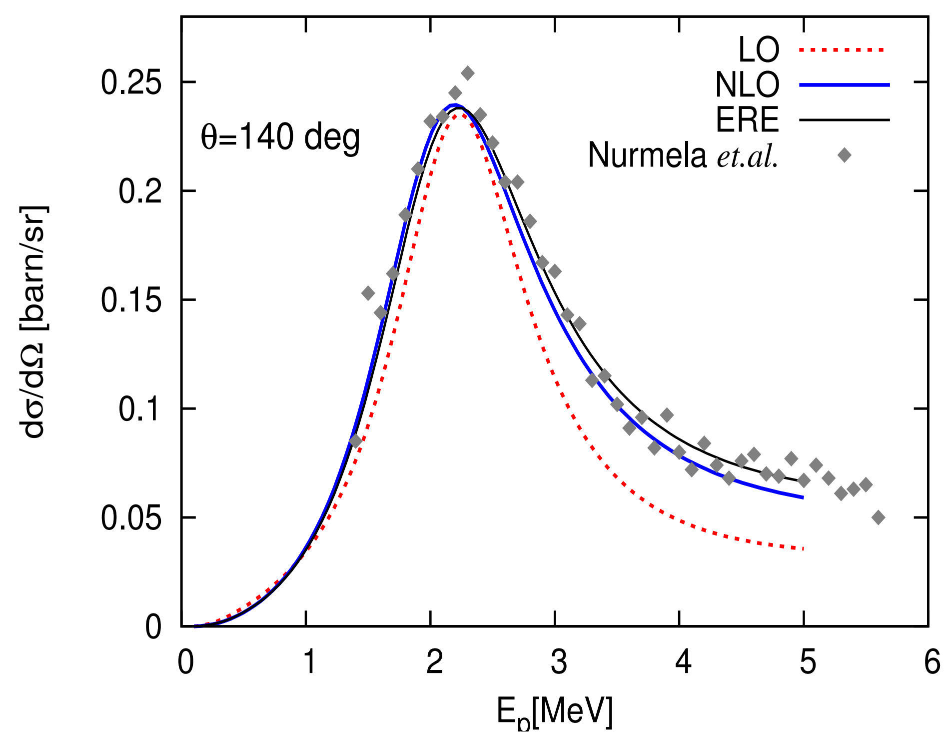
(RH, Bertulani, van Kolck, in preparation)





ab-initio: Nollett em et al., PRL 99, 022502 (2007),
Quaglioni & Navrátil, PRL 101, 092501 (2008)

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c/r^2+Coulomb: warm-up for 3α

- 3-body problem with large a $\sim$ 1D Schrödinger Eq. with $V(r) = 1/r^2$
- limit cycle for $c < -1/4$ $\Leftrightarrow$ Efimov spectrum
  (Beane *et al.*, Bawin and Coon, Braaten and Phillips, Long and van Kolck...)
- counterterm: log-periodic function of the cutoff
- Counterterm parameter Λ_*: iteration of quantum corrections
  (dimensional transmutation)
- model to study NR conformal invariance (Kaplan *et al.*)

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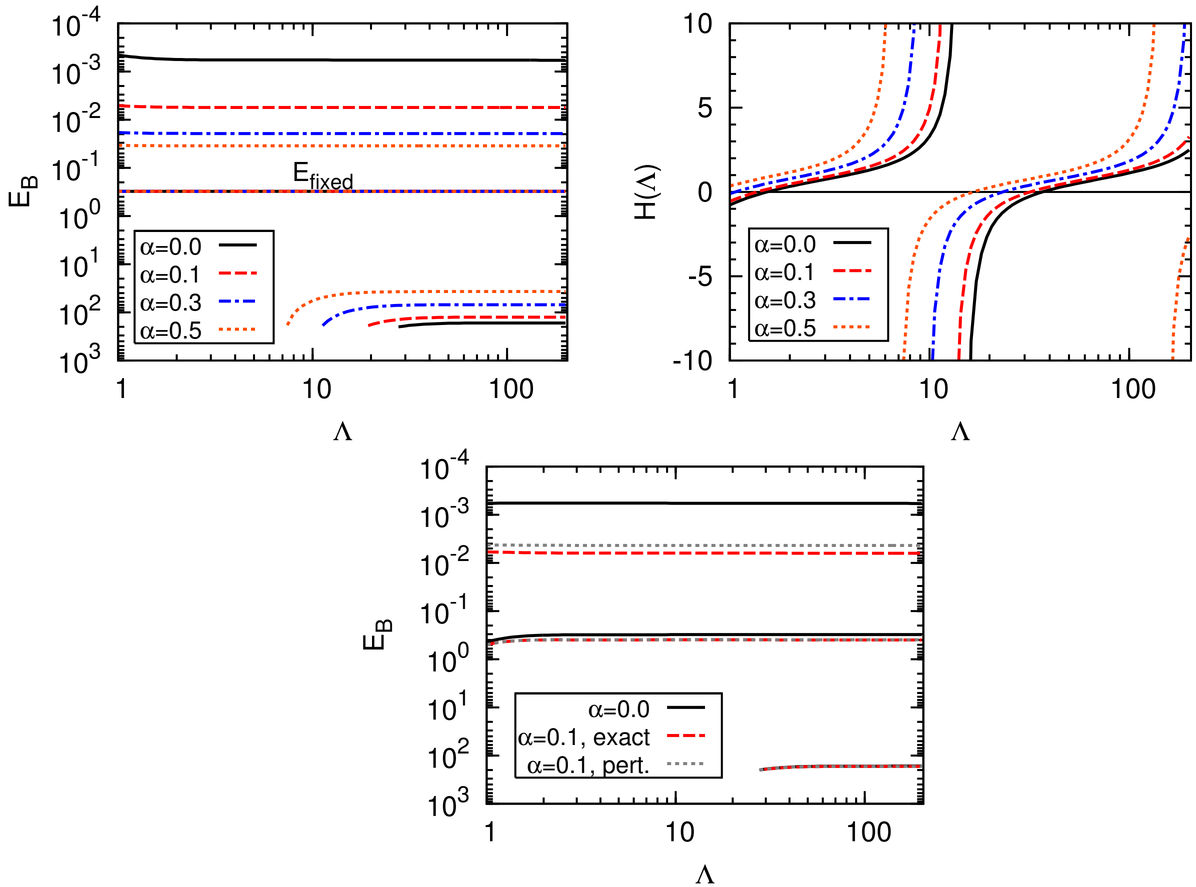
$1/r^2$+Coulomb: warm-up for 3α

(Hammer, RH, Eur. J. Phys. A 37, 193)



Summary

- Halo nuclei, cluster systems: promising area for halo/cluster EFT
- universality $\Leftrightarrow$ limit cycles
- $\alpha\alpha$ scattering
  - ⋆ Coulomb turned off $\Rightarrow$ conformal invariance @LO, Efimov spectrum in ^{12}C
  - ⋆ incredible amount of fine-tuning
  - ⋆ LO (parameter-free) works only at very low energies, NLO improves description up to $E_{LAB} \approx 3$ MeV
  - ⋆ extraction of the ERE parameters with improved errorbars
- p-α scattering: good description of the $P_{3/2}$ resonance
- future: 3α, $^{A}X(N,\gamma)^{A+1}Y$,p-^{7}Be, EM, Borromean halos, heavier nuclei, ...

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