Universality in QCD and Halo Nuclei

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The limits of existence of light nuclei, ECT* Trento, October 25-29, 2010

Agenda

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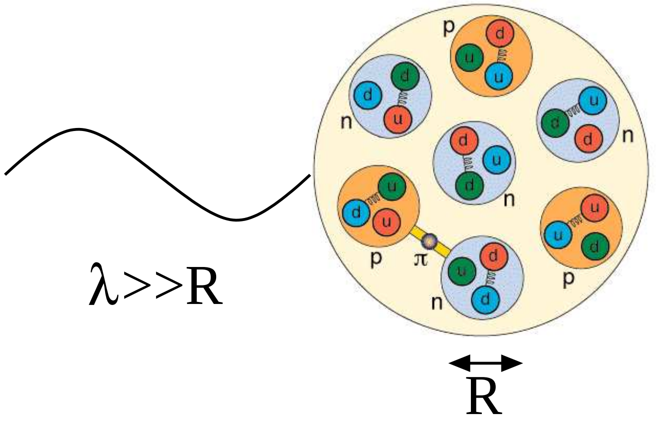
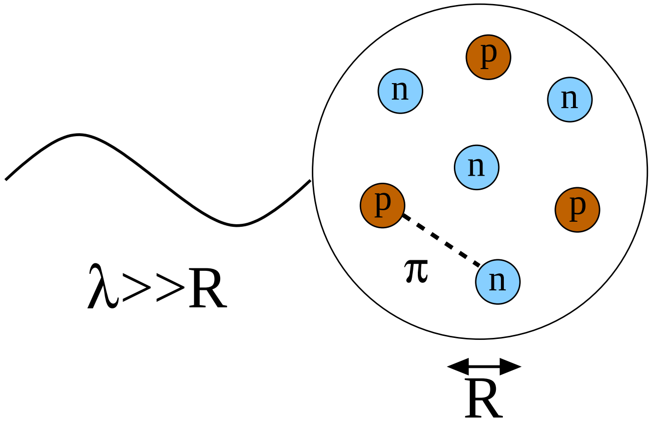
- Introduction
- Few-body Physics near Unitarity
- Effective Field Theory Framework
- Applications
  - EM properties of ^{11}Be (with D.R. Phillips)
  - Structure of two-neutron halo nuclei (with D. Canham)
- Summary and Outlook

Effective Theory

- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



Effective Theory



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  $\longrightarrow$ capture in low-energy constants using renormalization

  $\longrightarrow$ include long-range physics explicitly

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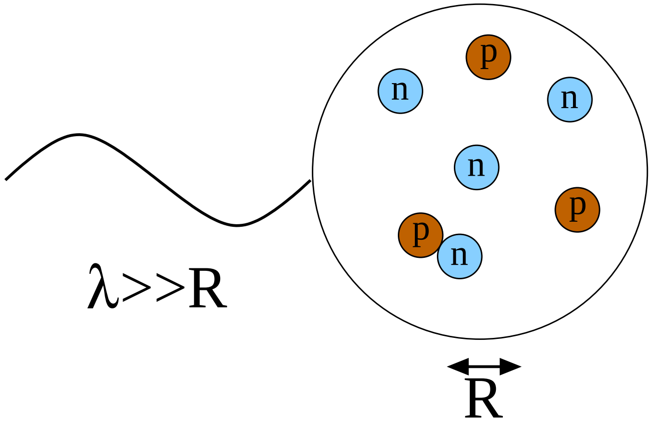
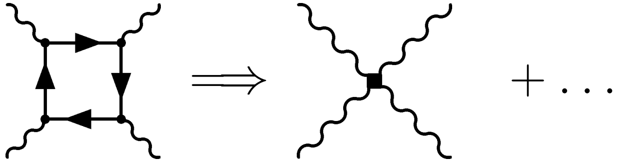


- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

Contact interactions for $\omega \ll m_e$:

$$\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \to \mathcal{L}_{eff}[A_\mu]$$



Few-Body Physics near Unitarity

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- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$T_2(k) = [k \cot \delta - ik]^{-1} \quad \Longrightarrow \quad i/k$$

- Scattering amplitude saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, perfect liquid, ...
- Use as starting point for EFT description of (effective) few-body systems with resonant interactions
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal properties

$$a > 0 \quad \Longrightarrow \quad B_d = \frac{1}{2\mu a^2} + \mathcal{O}(\ell/a)$$

Few-Body Physics near Unitarity

- Many physical systems are close to unitary limit
- Nuclear physics: NN-system, halo nuclei,...
  - ${}^1S_0$, ${}^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{11}Be $\Rightarrow$ ^{10}Be + n: n separation energy $\approx$ 0.5 MeV
- Particle physics:
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = (0.3 \pm 0.4) \text{ MeV}$$

- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ scattering length can be tuned using external magnetic field

Halo Nuclei

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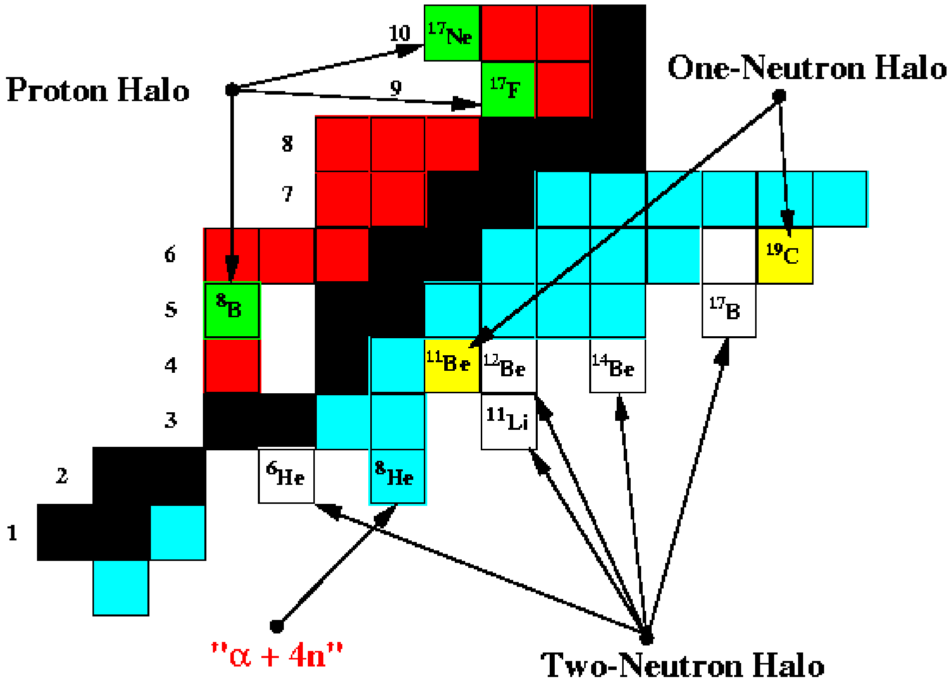
- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT



http://www.nupecc.org

- EFT for halo nuclei
  - $n\alpha$-System ("^{5}He") (Bedaque, Bertulani, HWH, van Kolck, 2002, 2003)
  - $\alpha\alpha$-System ("^{8}Be") (Higa, HWH, van Kolck, 2008)

Electromagnetic Structure of ^{11}Be

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- Properties of ^{11}Be
  - Ground state: $J^P = 1/2^+$, neutron separation energy: 504 keV
  - Excited state: $J^P = 1/2^-$, neutron separation energy: 184 keV
- Properties of ^{10}Be
  - Ground state: $J^P = 0^+$
  - First excitation: 3.4 MeV above g.s.

$\Rightarrow$ one neutron halo picture for ^{11}Be appropriate

- Separation of scales: $M_{lo}/M_{hi} \approx \frac{0.5}{3.5} = \frac{1}{7}$ $\quad \Rightarrow R_{core}/R_{halo} \approx 0.4$
- P-wave $n-^{10}$Be scattering volume: $a_1 = (457 \pm 67)$ fm^3

  Typel, Baur, Phys. Rev. Lett. **93** (2004) 142502
- Study EM properties in halo picture

  (cf. Typel, Baur, NPA **759** (2005) 247, EPJA **38** (2008) 355) universitätbonn

- Effective Lagrangian

(cf. Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003)

$$\begin{aligned}\mathcal{L} \; = \; & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\ & + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\ & - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \, i\overleftrightarrow{\nabla}_j \, c) + (c^\dagger \, i\overleftrightarrow{\nabla}_j \, n^\dagger) \pi_j \right] \\ & - \frac{g_1}{2} \frac{M-m}{M_{nc}} \left[\pi_j^\dagger \, i\overrightarrow{\nabla}_j \, (nc) - i\overleftrightarrow{\nabla}_j \, (n^\dagger c^\dagger) \pi_j \right] + \ldots\end{aligned}$$

- Parameters:
  - Leading order: g_0, Δ_1, $g_1 \Leftarrow B_0$, B_1, a_1 or $B(E1)(1/2^+ \to 1/2^-)$
  - Next-to-leading order: $\Delta_0 \Leftarrow B(E1)(1/2^+ \to 1/2^-)$ or dB/dE

Effective Field Theory for ^{11}Be

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- EFT generates S- and P-wave states from core-neutron contact interactions
- Reproduces correct asymptotics of wave functions for S- and P-wave states

$$
\begin{aligned}
u_0(r) &= A_0 \exp(-\gamma_0 r) \\
u_1(r) &= A_1 \exp(-\gamma_1 r) \left(1 + \frac{1}{\gamma_1 r}\right)
\end{aligned}
$$

- Focus on observables: no discussion of n-core interaction at short distances, spectroscopic factors, ...
- Halo EFT: expansion in R_{halo}/R_{core}
- Generating the S- and P-wave states in Halo EFT:

$\Longrightarrow$ sum the nc bubbles

Generating S- and P-Wave States

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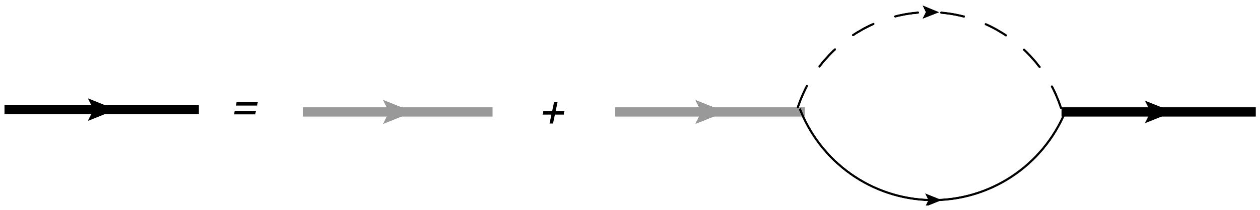
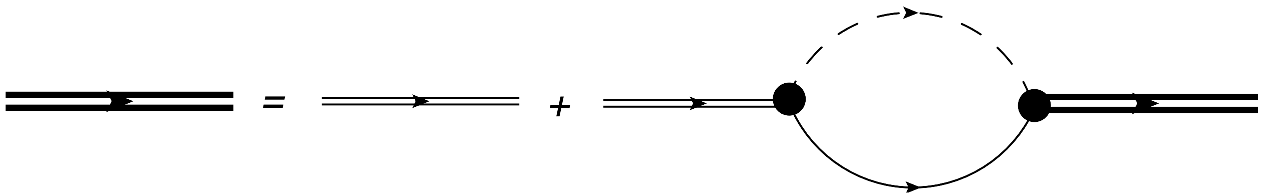
- S-wave state: $g_0 \sim R_{halo}$, nc loop $\sim 1/R_{halo} \Rightarrow$ sum bubbles
  (van Kolck, 1997, 1999; Kaplan, Savage, Wise, 1998)



- P-wave state:



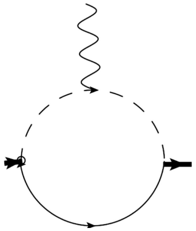
- Determine parameters from bound state pole and/or scattering parameters

$$D_\pi(p) \propto \frac{1}{r_1 + 3\gamma_1} \frac{1}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

where $\gamma_1 = \sqrt{2m_R B_1}$

Including Photons

- Minimal substitution: $\partial_\mu \to D_\mu = \partial_\mu + ie\hat{Q}A_\mu$
- S-wave form factor (LO):



$$G_c(|\mathbf{q}|) = \frac{2\gamma_0}{f|\mathbf{q}|}\arctan\left(\frac{f|\mathbf{q}|}{2\gamma_0}\right) \quad \text{where} \quad f = m_R/M = 1/11$$

- Charge radius of ^{11}Be: $\langle r_c^2\rangle_{^{11}\mathrm{Be}} = \langle r_c^2\rangle_{^{10}\mathrm{Be}} + \frac{f^2}{2\gamma_0^2}$
- Using the experimental value: $\langle r_c^2\rangle_{^{10}\mathrm{Be}} = 2.357(18)$ fm

  Nörteshäuser et al., Phys. Rev. Lett. **102** (2009) 062503
  - At LO: $\sqrt{\langle r_c^2\rangle_{^{11}\mathrm{Be}}} = 2.40$ fm
  - At NLO: $\sqrt{\langle r_c^2\rangle_{^{11}\mathrm{Be}}} = 2.44$ fm
- Experimental value: $\sqrt{\langle r_c^2\rangle_{^{11}\mathrm{Be}}} = 2.463(16)$ fm

  Nörteshäuser et al., Phys. Rev. Lett. **102** (2009) 062503

P-wave Form Factors

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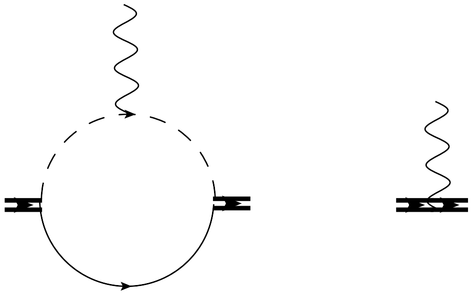
- P-wave form factor (LO):



(a) (b)

- Charge form factor:

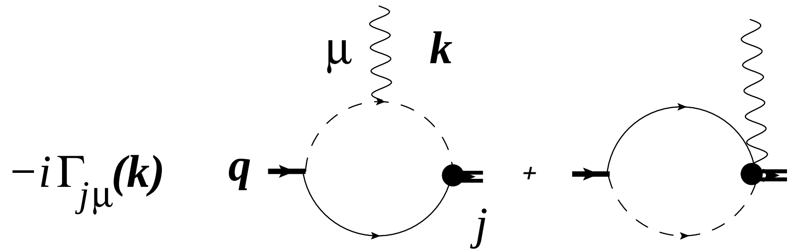
$$G_c(|\mathbf{q}|) = \frac{1}{r_1 + 3\gamma_1}\left[r_1 + \frac{1}{qf}\left(2qf\gamma_1 + (q^2f^2 + 2\gamma_1^2)\arctan\left(\frac{f|\mathbf{q}|}{2\gamma_1}\right)\right)\right]$$

- Quadrupole form factor

$$G_Q(|\mathbf{q}|) = \frac{2M_{nc}^2}{r_1 + 3\gamma_1}\frac{3}{4q^3f}\left(2qf\gamma_1 + (q^2f^2 - 4\gamma_1^2)\arctan\left(\frac{f|\mathbf{q}|}{2\gamma_1}\right)\right)$$

- No experimental information

S-to-P Transition



- Irreducible transition vertex

$$\Gamma_{ji} = \delta_{ji}\Gamma_E + k_j k_i \Gamma_M \qquad \text{for} \qquad \mathbf{k}\cdot\mathbf{q} = 0, \quad \mathbf{k}\cdot\epsilon = 0$$

- Current conservation: $k_\mu \Gamma_{j\mu} = 0 \implies \omega\Gamma_{j0} = k_j \Gamma_E$
- $B(E1)$ transition strength:

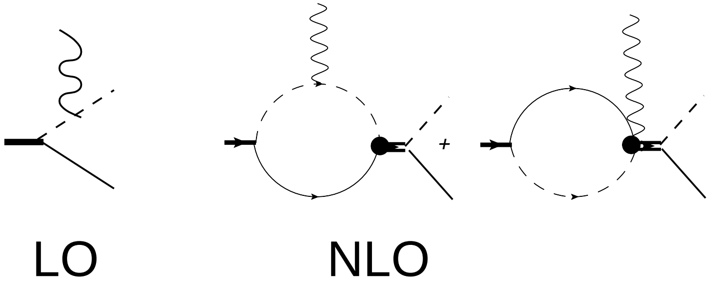
$$B(E1) = \frac{1}{4\pi}\left(\frac{\Gamma_E}{\omega}\right)^2 = \frac{Z_{eff}^2 e^2}{3\pi}\,\frac{\gamma_0}{-r_1}\left[\frac{2\gamma_1 + \gamma_0}{(\gamma_0+\gamma_1)^2}\right]^2 + \ldots$$

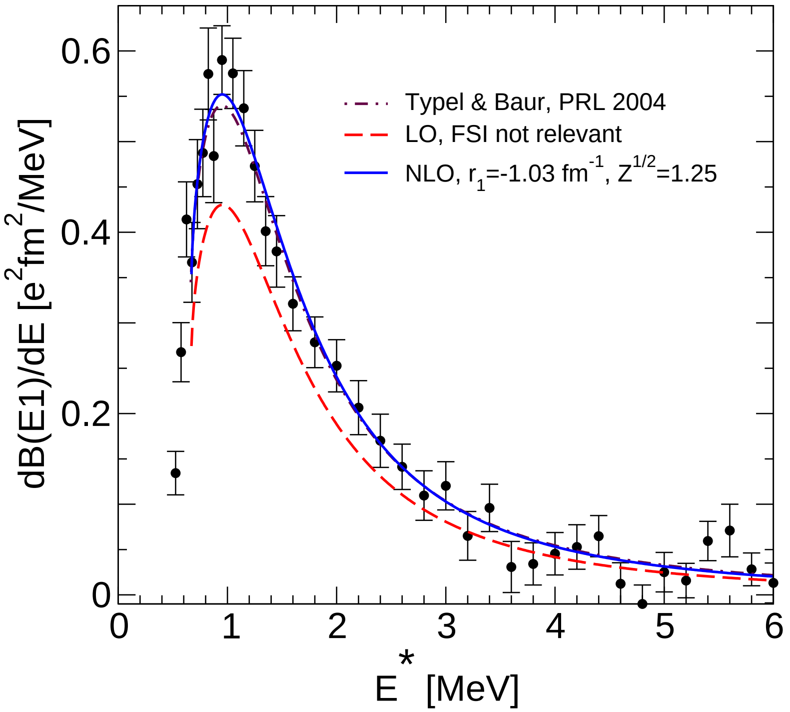
- No cutoff required: divergences cancel!
- Results: $B_{LO}(E1) = 0.130\ e^2$ fm^2, r_0 required for NLO
- Experiment: $B(E1) = 0.105\ldots0.116\ e^2$ fm^2

(Summers et al., PLB **650** (2007) 124; Millener et al., PRC **28** (1983) 497)

Coulomb Dissociation

- Transition to the continuum:





- Reasonable convergence
- At LO: no FSI
- At NLO: $r_1 = -1.03$ fm^{-1} & $\sqrt{Z_\sigma} = 1.25$
- Detector resolution folded in

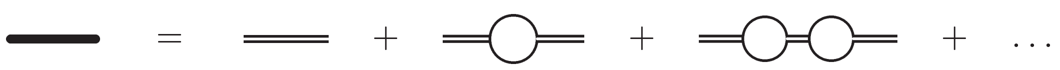
Data: Palit et al., PRC **68** (2003) 034318

Three-Body System

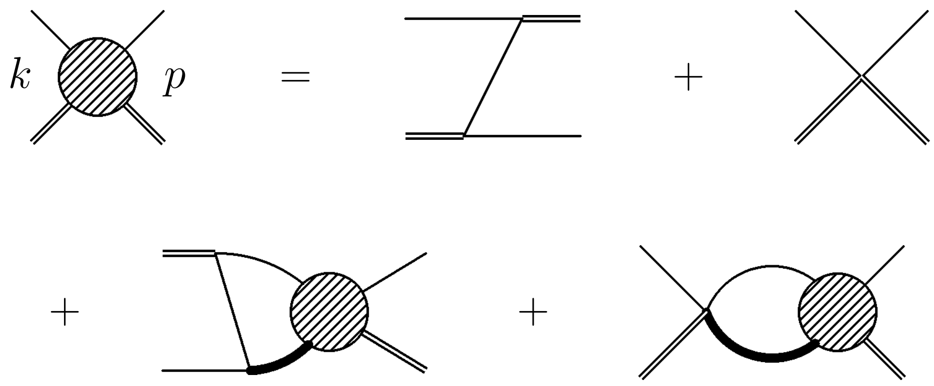
- Effective Lagrangian (Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$

- 2- and 3-body interaction at leading order: g_2, g_3 enhanced!
- 2-body amplitude:



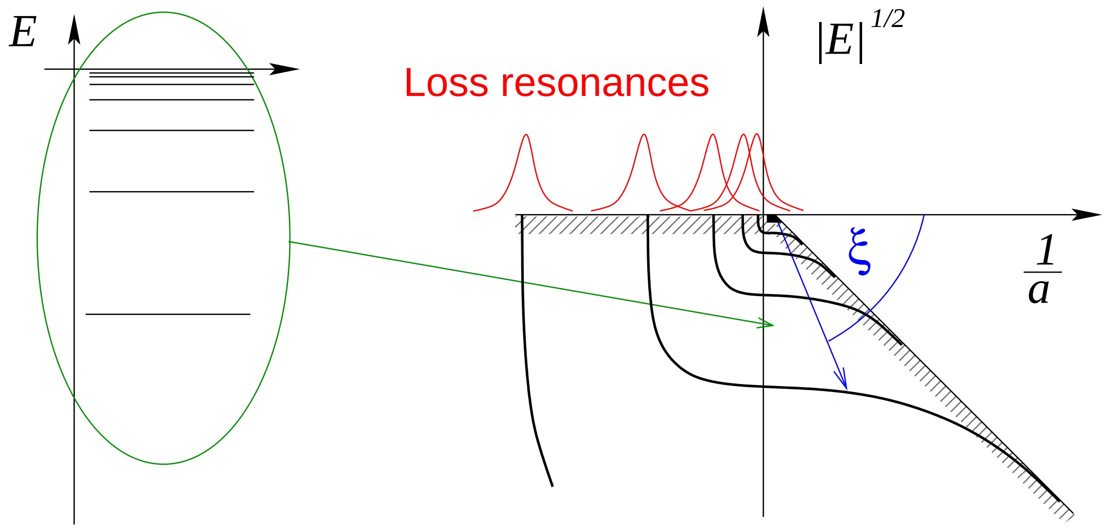
- 3-body amplitude:



- 3-body coupling: $\Lambda^2 g_3(\Lambda)$ periodic $\Longrightarrow$ limit cycle

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states
  (V. Efimov, Phys. Lett. **33B** (1970) 563)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad \left(e^{\pi/s_0}\right)^2 = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Four-Body System

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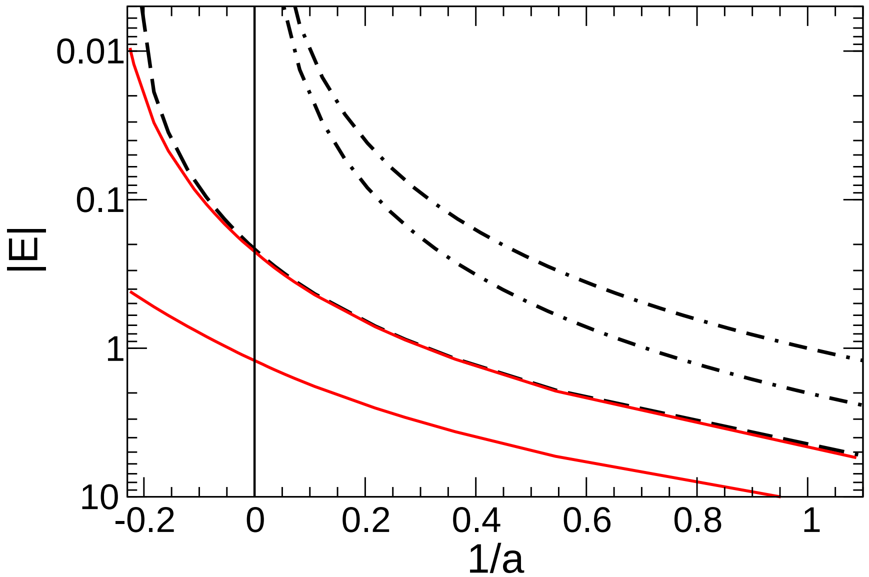
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)

4-body "Efimov-plot":

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



- Extension to thresholds, widths

  von Stecher, D'Incao, Greene, Nature Physics **5** (2009) 417; Deltuva, arXiv:1009.1295

- Signature in Cs loss data

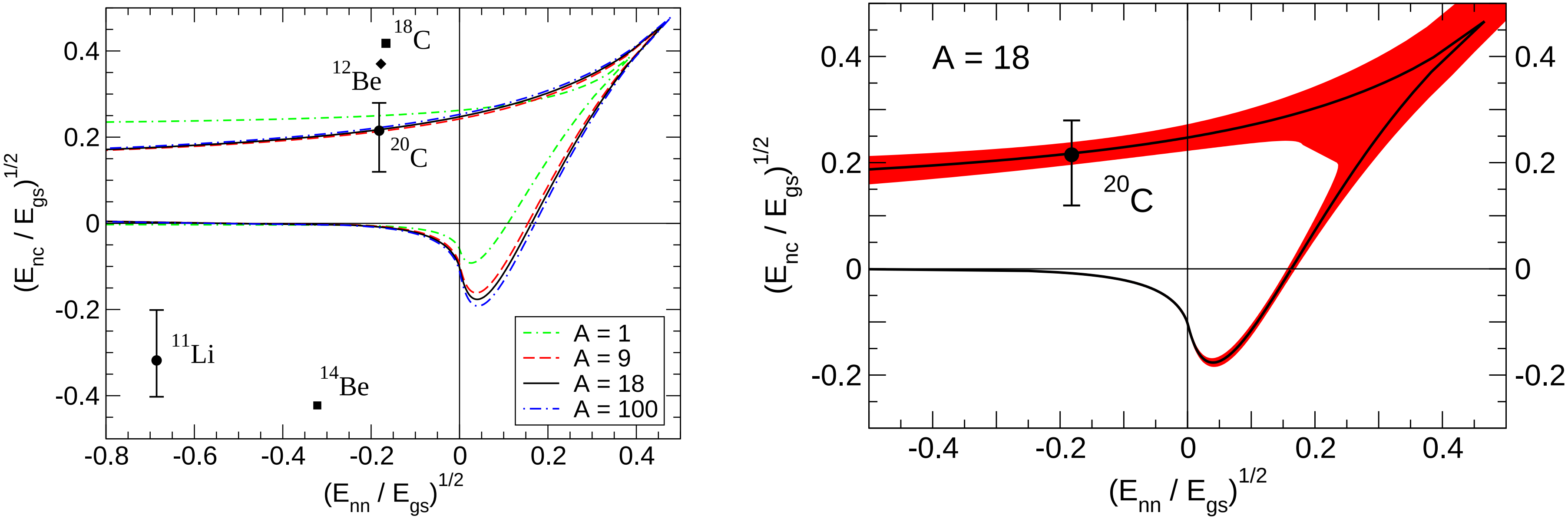
  Ferlaino, Knoop, Berninger, Harm, D'Incao, Nägerl, Grimm, PRL **102** (2009) 140401

- Four-body parameter very close to resonance?

  Yamashita, Tomio, Delfino, Frederico, EPL **75** (2006) 555

3-Body Halos

- Examples: ^{14}Be $\longleftrightarrow$ ^{12}Be +n +n, ^{20}C $\longleftrightarrow$ ^{18}C +n +n
- "Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of "core"
- Efimov effect in halo nuclei? $\Rightarrow$ excited states



Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Fedorov, Jensen, Riisager, 1994; Amorim, Frederico, Tomio, 1997)

- Unchanged at NLO if $r_{nc} \lesssim 1/M_\pi$ (Canham, HWH, 2010)

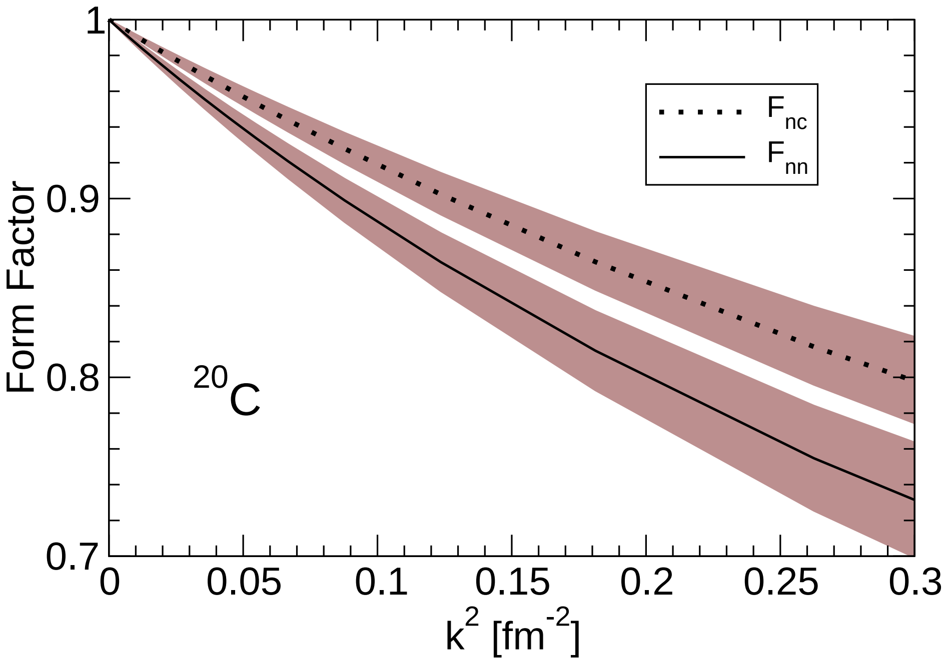
Form Factors and Radii

- Structure of halo nuclei $\to$ matter form factors, radii

$$F(k^2) = \int d^3p\, d^3q\, \Psi(\vec{p},\vec{q})\Psi(\vec{p},\vec{q}-\vec{k}) = 1 - \frac{1}{6}k^2\langle r^2\rangle + \ldots$$

- Candidate: ^{20}C
- Form factors:

  F_{nn} neutron-neutron
  F_{nc} neutron-core



Canham, HWH, Eur. Phys. J. A **37** (2008) 367

Form Factors and Radii

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- Structure of halo nuclei $\rightarrow$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]
^{14}Be	1120	-200.0	4.1 ± 0.5	3.5 ± 0.5
^{20}C	3506	161	2.8 ± 0.3	2.4 ± 0.3
	3506	530	3.0 ± 0.7	2.5 ± 0.6
	3506	60	2.8 ± 0.2	2.3 ± 0.2
^{20}C*	65 ± 6.8	60	42 ± 3	38 ± 3

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\rightarrow \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm
  (Marques et al., Phys. Rev. C **64** (2001) 061301)
- Unchanged at NLO if $r_{nc} \lesssim 1/M_\pi$ (Canham, HWH, 2010)

Summary and Outlook

- EFT for systems close to the unitarity limit
  $\Longrightarrow$ large scattering length
  - Controlled, systematic approach $\Longrightarrow$ error estimates
  - Straightforward inclusion of external currents
- Universality $\Rightarrow$ theory for ^{11}Be applicable to any $1n$ halo nucleus with shallow S- and P-wave state
- Universal theory has many applications
  - ultracold atoms
  - light nuclei, halo nuclei
  - hadronic molecules
- Efimov effect in nuclear physics?
  - difficult to identify uniquely
  - But: limit cycle in QCD with unphysical quark masses?
    (Braaten, HWH, Phys. Rev. Lett. **91** (2003) 102002)

Additional Slides

Renormalization

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- Observables are independent of regulator/cutoff Λ

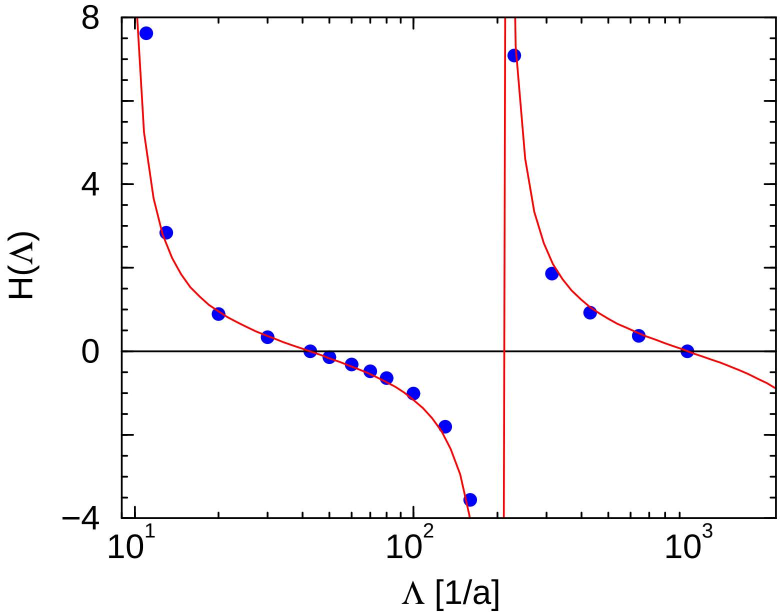
$\Rightarrow$ Running coupling $H(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup



$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow$ 3-body observable: B_t, K_3,...