Study of the 3N force in $A = 4$ systems

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INFN, Sezione di Pisa

ECT* workshop
"The limits of existence of light nuclei"
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Outline

1. Study of the 3N force in $A = 3, 4$
2. Extension of the HH method to $A > 4$

Collaborators

- A. Kievsky, L. Girlanda, L.E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
- M. Gattobigio - *INL, Nice (France)*

NN & 3N interaction

NN potentials

- ”Old models”: Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (INOY [Doleschall, 2008])
- Effective field theory
  - J-N3LO [Epelbaum and Coll, 1998-2006]
  - I-N3LO [Entem & Machleidt, 2003]

- Low-q interaction [Bogner and Coll., 2001-2007]
- UCOM interaction [Roth and Coll. 2004-2010]
- SRG interaction [Furnstahl and Coll.,2008-2010]

3N potentials

- “Old models”: Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]; Brazil [Robilotta & Coelho, 1986]; Urbana [Pudliner *et al*, 1995]
- Effective field theory
  - J-N2LO [Epelbaum *et al*, 2002]
  - N-N2LO [Navratil, 2007]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, Δ, CSB, . . .

HH method (1)

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \ldots$$

Search for accurate solution of $H\Psi = E\Psi$

- Expansion of Ψ on the basis of Hyperspherical Harmonics
- Problems: 1) convergence 2) antisymmetrization of the basis 3) boundary conditions for scattering states, ...
- Accurate, state-of-the-art, calculations of bound and elastic observables
- Treatment of non-local or projecting potential possible
- Hard-core potential $\rightarrow$ inclusion of a correlation factor
- Still to be solved: proper treatment of breakup channels ($N + d \rightarrow N + n + p$)

A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states, A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, and L. Girlanda J. Phys. G, **35**, 063101 (2008)

The HH method (2)

HH functions

- hyperradius $\rho^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\xi_1}{\rho}, \ldots, \frac{\xi_{A-1}}{\rho}\}$ (ξ_i Jacobi vectors)
- $T = T_\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

$$\Phi_k = L_m^{(3A-4)}(\beta\rho)e^{-\beta\rho/2}\,\mathcal{A}\left[\mathcal{Y}_{[K]}(\Omega)\right]$$

Advantages

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- coordinate/momentum space interaction

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Scattering calculation

Example: $A - B$ elastic scattering

$$\Omega^F_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}$$

$$\Omega^G_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

$$\Omega^\pm_{LS}(A,B) = \Omega^G_{LS}(A,B) \pm i\Omega^F_{LS}(A,B)$$

$$|\Psi_{LS}\rangle = \sum_k a_{LS,k}\Phi_k + |\Omega^F_{LS}(p, {}^3\mathrm{He})\rangle + \sum_{L'S'} T_{LS,L'S'} |\Omega^+_{L'S'}(p, {}^3\mathrm{He})\rangle$$

- $T_{LS,L'S'}$ = T-matrix elements
- $a_{LS,n}$ and $T_{LS,L'S'}$ determined using the Kohn variational principle (KVP)

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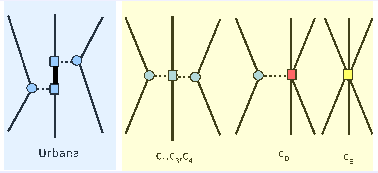
3N forces



- N-N2LO:
  - 2π-exchange: c_1, c_3, c_4 (taken from NN)
  - OPE+contact: c_D, c_E fitted ($^3\mathrm{H}$, $^4\mathrm{He}$ B.E.)
- Urbana:
  - 2π-exchange (FM): $a_{2\pi}$, $c_{2\pi} = a_{2\pi}/4$
  - repulsive term: U_0
- TM/Brazil:
  - 2π-exchange: a, b, d (taken from πN), Λ cutoff (fitted to $^3\mathrm{H}$ B.E.)

Study of the 3N force (1)

Potential	$B(^3\mathrm{H})$ (MeV)	$B(^4\mathrm{He})$ (MeV)	$^2a_{nd}$ (fm)
AV18	7.624	24.22	1.258
I-N3LO	7.854	25.38	1.100
AV18/TM'	8.440	28.31	0.623
AV18/UIX	8.479	28.48	0.578
I-N3LO/N-N2LO	8.474	28.37	0.675
Exp.	8.482	28.30	$0.645 \pm 0.003 \pm 0.007$

n-d zero-energy scattering= first "excited" state of $^3\mathrm{H}$

Fix the parameters of the 3N force ($\sim$ 5):

1. $^3\mathrm{H}$ and $^4\mathrm{He}$ binding energies
2. $n-d$ doublet scattering lenght ($J = 1/2$)
3. $n-d$ and $p-d$ elastic scattering (A_y)

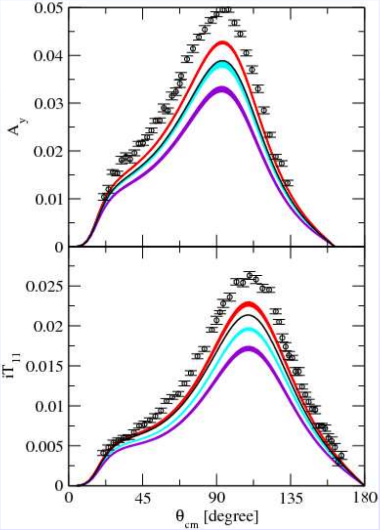
Study of the 3N force (2)

Urbana $$W(1,2,3) = aW^{a}_{2\pi}(1,2,3) + cW^{c}_{2\pi}(1,2,3) + U_0 W_R(1,2,3)\ .$$

Potential	a	c/a	U_0	$B(^3\mathrm{H})$ (MeV)	$B(^4\mathrm{He})$ (MeV)	$^2a_{nd}$ (fm)
AV18				7.624	24.22	1.258
AV18+URIX	-0.0293	0.25	0.0048	8.479	28.48	0.578
AV18+URIX-1	-0.0200	1.625	0.0176	8.484	28.33	0.644
AV18+URIX-2	-0.0250	1.25	0.0182	8.484	28.34	0.644
AV18+URIX-3	-0.0293	1.00	0.0181	8.484	28.33	0.643
Exp.				8.482	28.30	0.645±0.003±0.007

- Significative modifications of the 3N force
- We have found 3 families of the 3N force: the N-N2LO, Urbana, and TM- families
- The TM/BRazil must be supplement by a repulsive part
- They differ in the short-range part (regularization)

Study of the 3N force (3)



p-d elastic scattering 3 MeV: A_y and iT_{11}

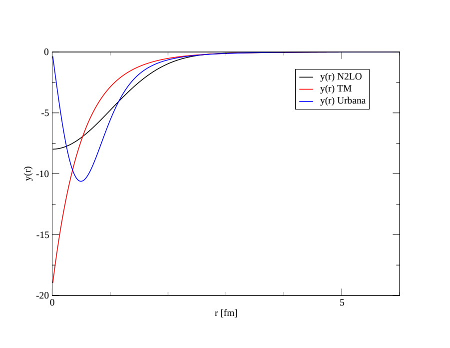
$N-d$ elastic scattering
solid curve: AV18+UIX
red band= AV18+N-N2LO-family
cyan band= AV18+Urbana-family
violet band= AV18+TM-family

work in progress

Differences between N2LO & Urbana TNI

Radial dependence

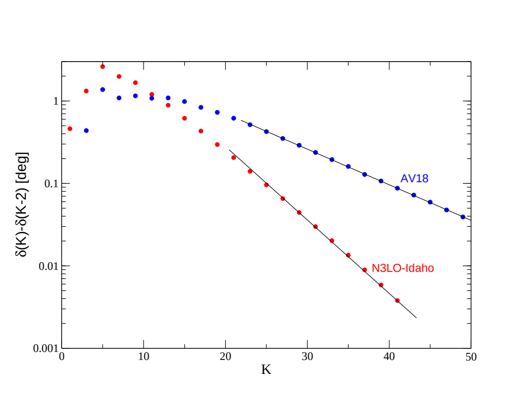
$$W(1,2,3) = c_{2\pi}\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3,\}\{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3,\}y(r_{12})y(r_{13}) + \ldots$$



- Study different radial dependence (regularization) in progress

$A = 4$ scattering

$n - {}^3\mathrm{H}$ at $E_{Lab} = 4$ MeV, ${}^3P_2$ wave



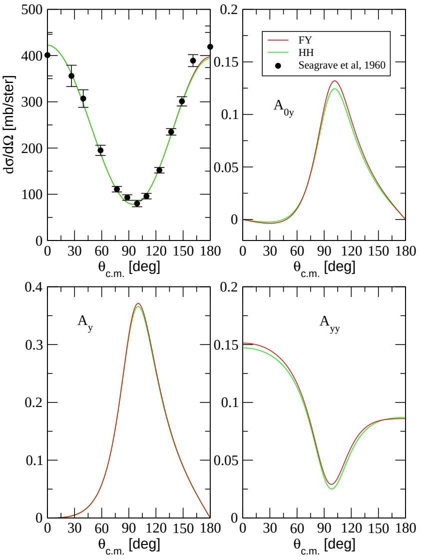
Convergence with the grand-angular quantum number

$n - {}^3\mathrm{H}$ elastic scattering

Comparison with the FY calculation by Deltuva & Fonseca, 2007

N3LO-Idaho potential, $E_{\mathrm{n}} = 3.5$ MeV

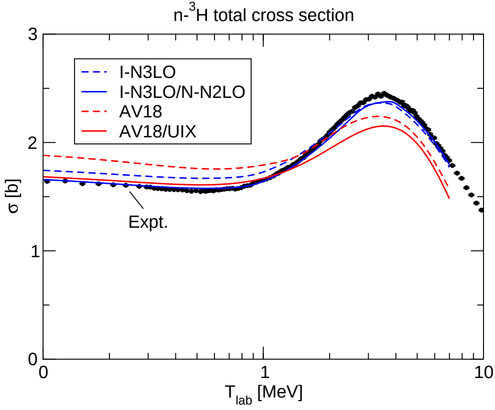
Phase-shift	HH	FY	Phase-shift	HH	FY
${}^1S_0$	-65.68	-65.66	${}^3P_0$	20.23	20.21
${}^3S_1$	-58.03	-58.20	${}^1P_1$	20.70	20.90
${}^3D_1$	-0.92	-0.92	${}^3P_1$	40.80	40.98
ϵ	0.71	0.68	ϵ	9.99	9.57
${}^1D_2$	-0.84	-0.82	${}^3P_2$	43.77	43.58
${}^3D_2$	-1.59	-1.45	${}^3F_2$	0.07	0.05
ϵ	2.49	2.62	ϵ	1.17	1.14



I-N3LO potential
$E_n = 3.5$ MeV

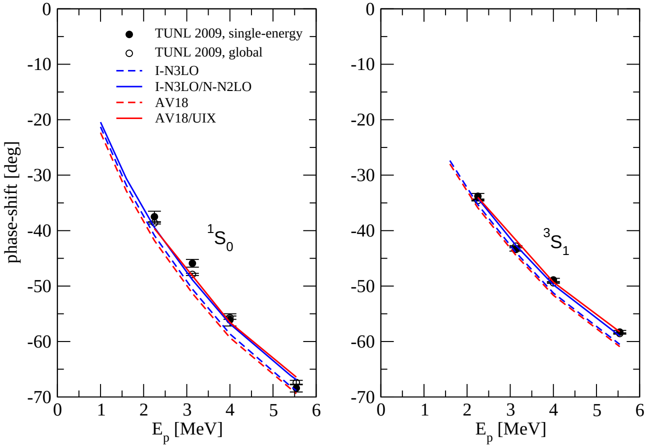
FY= Deltuva & Fonseca, 2007

$n - {}^3\mathrm{H}$ total cross section



$p - {}^3\mathrm{He}$ phase-shift - S-waves

Comparison with the recent PSA by Daniels & Clegg, TUNL (2010)



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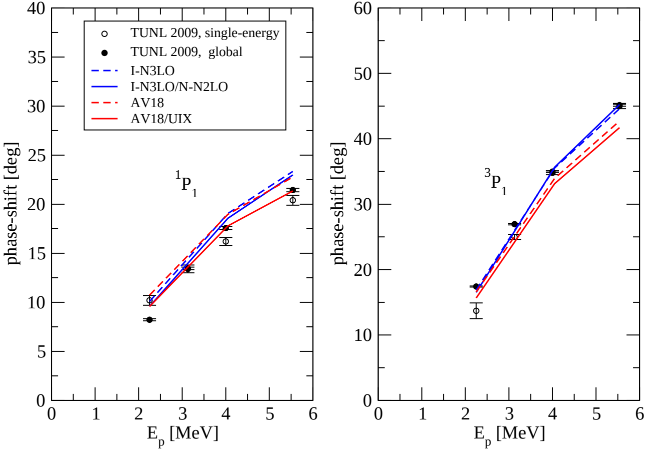

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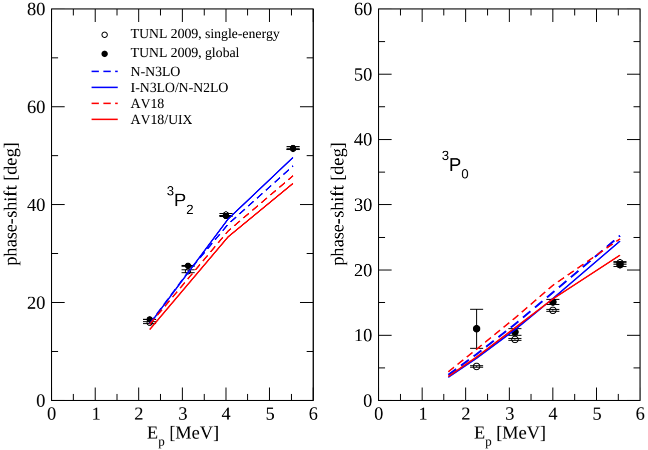

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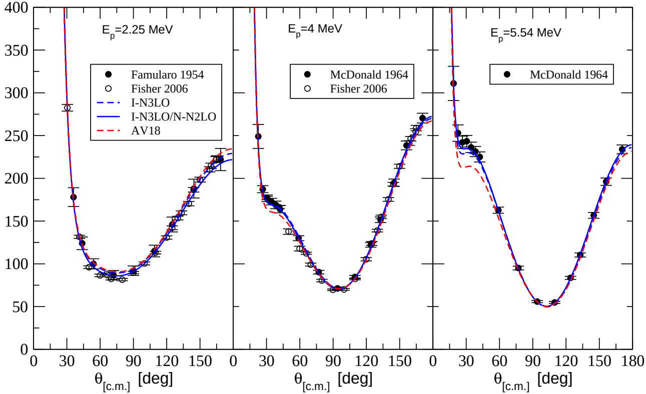
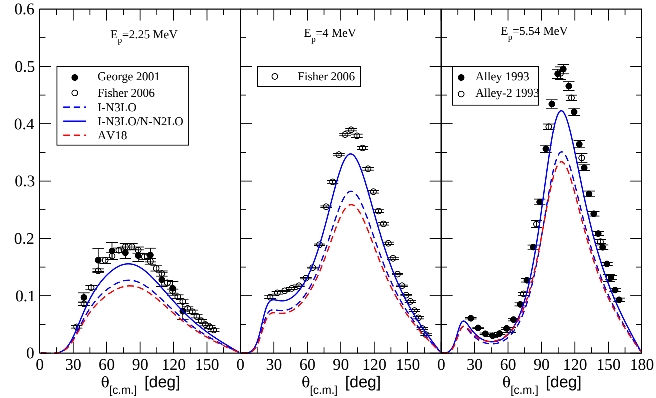
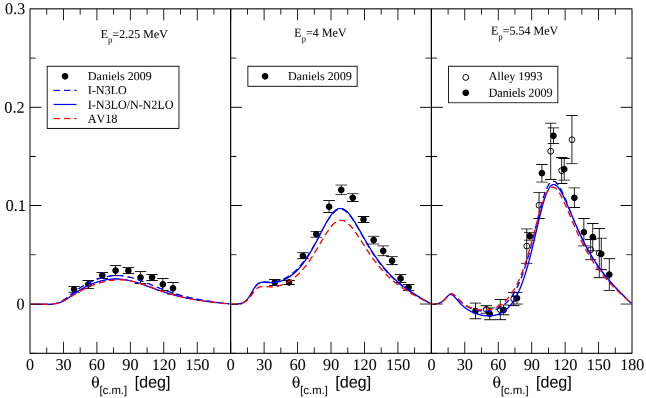


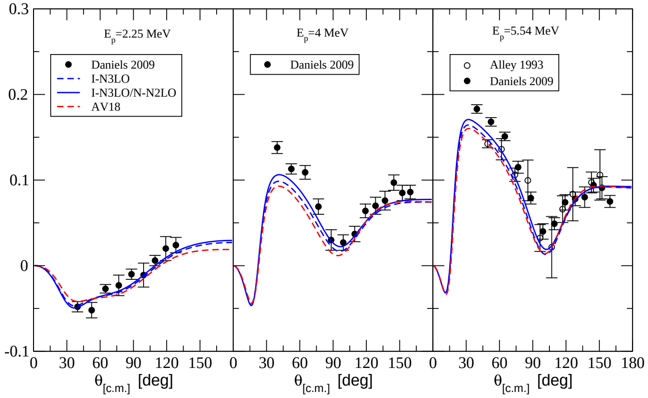
$p - {}^3\mathrm{He}\ A_y$



$p - {}^3\mathrm{He}\ A_{0y}$



$p - {}^{3}\mathrm{He}\ A_{yy}$



$n - {}^3\mathrm{He}$ scattering lenghts

$$|\Psi_{LS}\rangle = \sum_k a_{LS,k}\Phi_k + |\Omega^F_{LS}(n,{}^3\mathrm{He})\rangle + \sum_{L'S'} T^{el}_{LS,L'S'}|\Omega^+_{L'S'}(n,{}^3\mathrm{He})\rangle + \sum_{L'S'} T^{ex}_{LS,L'S'}|\Omega^+_{L'S'}(p,{}^3\mathrm{H})\rangle$$

-
  - $\gamma = 1 \equiv p - {}^3\mathrm{H}$
  - $\gamma = 2 \equiv n - {}^3\mathrm{He}$
- $\mathcal{S}^{\gamma,\gamma'}_{LS,L'S'}$: S-matrix elements

$$a_S = \lim_{q_{n^3\mathrm{He}}\to 0} \frac{1 - \mathcal{S}^{2,2}_{0S,0S}}{2iq_{n^3\mathrm{He}}}$$

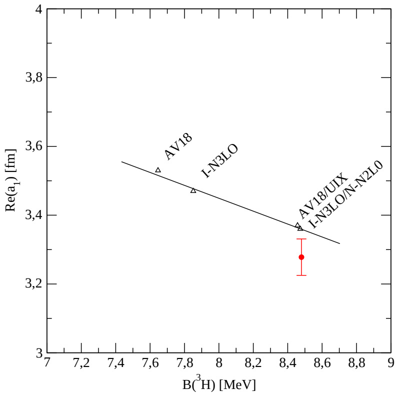
- since channel $1 \equiv p - {}^3\mathrm{H}$ is open, $|\mathcal{S}^{2,2}_{0S,0S}| < 1$ and a_S is complex.
- $\sigma(n + {}^3\mathrm{He} \to p + {}^3\mathrm{H}) \propto [-\mathrm{Im}a_0 - 3\mathrm{Im}a_1]$

$n - {}^3\mathrm{He}$ scattering lengths [fm]

Int.	Method	a_0 (fm)	a_1 (fm)
AV18	HH	$7.69 - i5.70$	$3.56 - i0.0077$
	RGM	$7.79 - i4.98$	$3.47 - i0.0066$
	FY	$7.71 - i5.25$	$3.43 - i0.0082$
AV18/UIX	HH	$7.89 - i3.44$	$3.39 - i0.0059$
	RGM	$7.63 - i4.05$	$3.31 - i0.0051$
I-N3LO	HH	$7.57 - i4.97$	$3.46 - i0.0048$
	FY		$3.56 - i0.0070$
	AGS	$7.82 - i4.51$	$3.47 - i0.0068$
I-N3LO/N-N2LO	HH		$3.37 - i0.0042$
Exp.[ILL-1]		$7.370(58) - i4.448(5)$	$3.278(53) - i0.001(2)$
Exp.[ILL-2]		7.46(2)	3.36(1)
Exp.[NIST]		7.57(3)	3.48(2)

RGM: Hofmann & Hale, PRC **77**, 044002 (2008)
FY: Lazauskas, `nucl-th arXiv:0905.3119`
ILL-1: Zimmer *et al.*, EPJA **4**, 1 (2002)
ILL-2: Ketter *et al.*, EPJA **27**, 243 (2006)
NIST: Huffman *et al.*, PRC **70**, 014004 (2004)

Triplet $n - {}^3\mathrm{He}$ scattering lenght vs. $B({}^3\mathrm{H})$



Also calculated by Deltuva & Fonseca, (2007)

Conclusion of the first part

1. Preliminary "fit" of the 3N force parameters (N2LO)
   - $^3\mathrm{H}$, $^4\mathrm{He}$ B.E., $n-d$ doublet scattering lenght
   - $N-d$ A_y puzzle still there
2. Work in progress: study extended to
   - $p-d \rightarrow p+p+n$ (breakup: $d^5\sigma/d\Omega_1 d\Omega_2 dS$)
   - $A=4$ scattering

Non-symmetrical basis

$\Psi = \sum_k a_k \Phi_k$

- It would be easy to use states $\widetilde{\Phi}_k$ constructed without any particular symmetry
- In fact: to construct antisymmetric states very difficult as A increases
  $[\Phi_k = \sum_{k'} b_{k'} \widetilde{\Phi}_k]$
- Idea: solve $H \sum_k a_k \widetilde{\Phi}_k = E \sum_k a_k \widetilde{\Phi}_k$
- since H is a symmetric operators, the eigenstates are also eigenstates of the symmetric group S_A
- Last step: select the antisymmetrical eigenstates
  - multiplicity
  - diagonalizing some operator

"transposition operator" $[(2)] = \sum_{i<j} (i \leftrightarrow j)$

A-body HH functions

HH functions

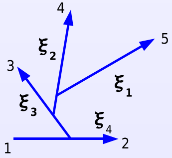
- Jacobi vectors ($N = A - 1$)

$$\boldsymbol{\xi}_{N-j+1} = \sqrt{\frac{2j}{j+1}}\left(\mathbf{r}_{j+1} - \frac{1}{j}\sum_{k=1}^{j}\mathbf{r}_k)\right)$$

- hyperradius $\rho^2 = \xi_1^2 \cdots + \xi_N^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\hat{\boldsymbol{\xi}}_1, \ldots, \hat{\boldsymbol{\xi}}_N, \phi_2, \ldots, \phi_N\}$

$$\cos\phi_j = \frac{\xi_j}{\sqrt{(\xi_1)^2 + \cdots + (\xi_j)^2}} \qquad k = 2, \ldots, N$$

- HH functions $\mathcal{Y}_{[K]}^{LM,K}(\Omega)$
- $\rho^K \mathcal{Y}_{[K]}^{LM,K}(\Omega)$ harmonic polynomials of degree K

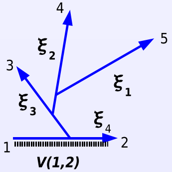


Expansion basis

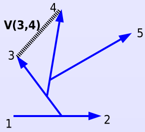
$$\widetilde{\Phi}_{m[K]}(\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_N) = g_m(\rho)\mathcal{Y}_{[K]}^{LM,K}(\Omega)$$

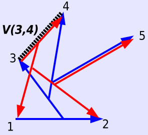
Matrix element calculation

$$\langle \widetilde{\Phi}_{m',[K']}(\xi_1,\ldots,\xi_N)| \sum_{i<j} V(i,j) | \widetilde{\Phi}_{m,[K]}(\xi_1,\ldots,\xi_N)\rangle$$



$$V^{(1,2)}_{m'[K'],m[K]} \sim \delta_{l_1,l'_1}\cdots\delta_{l_N,l'_N}\cdots\int d\phi_N\, d\rho g_{m'}(\rho)P(\phi_N)V(\rho\cos\phi_N)g_m(\rho)$$





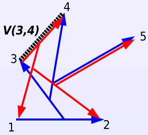
Transformation coefficients

- Ω' new hyperangles obtained with the "alternative" Jacobi vectors

$$\mathcal{Y}^{LM,K}_{[K]}(\Omega') = \sum_{[K']} C^{LM}_{[K][K']}\mathcal{Y}^{LM,K}_{[K']}(\Omega)$$

- L & K conserved in the transformation
- $C^{LM,K}_{[K][K']}$ are usually very large matrices

Transformation coefficients



"Transpositions"

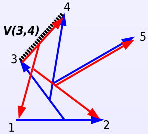
- $(1,2,3,4,5) \rightarrow (3,4,1,2,5)$ realized in "steps"
  - $(1,2,3,4,5) \rightarrow (1,3,2,4,5)$
  - $(1,3,2,4,5) \rightarrow (3,1,2,4,5)$
  - $(3,1,2,4,5) \rightarrow (3,1,4,2,5)$
  - $(3,1,4,2,5) \rightarrow (3,4,1,2,5)$
- "transposition j": exchange of particles $j-1, j$

Example:transposition $3 \rightarrow 4$

$$\begin{aligned}
\boldsymbol{\xi}_4 &= \boldsymbol{r}_2 - \boldsymbol{r}_1 & &\rightarrow \boldsymbol{\xi}'_4 = \boldsymbol{\xi}_4 \\
\boldsymbol{\xi}_3 &= \sqrt{\tfrac{4}{3}}\left(\boldsymbol{r}_3 - \tfrac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right) & &\rightarrow \boldsymbol{\xi}'_3 = \tfrac{1}{3}\boldsymbol{\xi}_3 + \tfrac{2\sqrt{2}}{3}\boldsymbol{\xi}_2 \\
\boldsymbol{\xi}_2 &= \sqrt{\tfrac{6}{4}}\left(\boldsymbol{r}_4 - \tfrac{\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3}{3}\right) & &\rightarrow \boldsymbol{\xi}'_2 = \tfrac{2\sqrt{2}}{3}\boldsymbol{\xi}_3 - \tfrac{1}{3}\boldsymbol{\xi}_2 \\
\boldsymbol{\xi}_1 &= \sqrt{\tfrac{8}{5}}\left(\boldsymbol{r}_5 - \tfrac{\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4}{4}\right) & &\rightarrow \boldsymbol{\xi}'_1 = \boldsymbol{\xi}_1
\end{aligned}$$

- $\xi_2^2 + \xi_3^2 = (\xi'_2)^2 + (\xi'_3)^2$
- only two hyperangles change
- the corresponding coefficients can be obtained by a three-dimensional integration

Calculation of the eigenvalues



$$\mathcal{C}^{ij,LM}_{[K][K']} = \left[\mathcal{A}^{LM}_{i_1} \cdots \mathcal{A}^{LM}_{i_n}\right]_{[K][K']}$$

$$\langle\widetilde{\Phi}_{m'[K']}(\xi_1,\ldots,\xi_N)|\sum_{i<j} V(i,j)|\widetilde{\Phi}_{m[K]}(\xi_1,\ldots,\xi_N)\rangle = \sum_{i<j} \mathcal{A}^{LM}_{i'_1}\cdots\mathcal{A}^{LM}_{i'_n} V^{(1,2)} \mathcal{A}^{LM}_{i_1}\cdots\mathcal{A}^{LM}_{i_n}$$

- $\{H\}$ = Hamiltonian in this basis
- Lanczos techniques $X_{n+1} = \{H\}X_n$: $\mathcal{A}^{LM}_i X$ & $V^{(1,2)}X$ very economic
- the problem scales approximately with A

- Application for $A = 5, 6$ with a central potential (Volkov)- no spin/isospin
- Problem: search the eigenvectors with the right symmetry
- Symmetry -adapted Lanczos technique: since $\{H\}$ is a symmetric operator, starting with a vector with a given symmetry, only the eigenvectors of such symmetry are generated (projection needed due to round errors)

$A = 5$

$$[(2)] = \sum_{i<j} (i \leftrightarrow j)$$

$$V^{(1,2)} = (-)^{\ell_N}$$

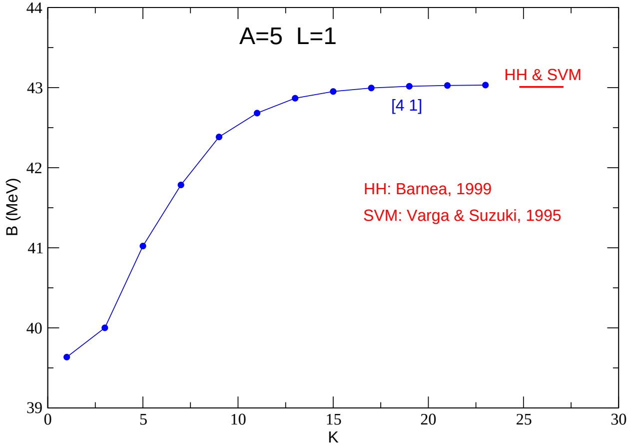
irreps of S_5	Multiplicity	[(2)]
[5]	1	10
[4, 1]	4	5
[3, 2]	5	2
[3, 1, 1]	6	0
[2, 2, 1]	5	− 2
[2, 1, 1, 1]	4	− 5
[1, 1, 1, 1, 1]	1	−10

Eigenvalues from $H\Psi = E\Psi$

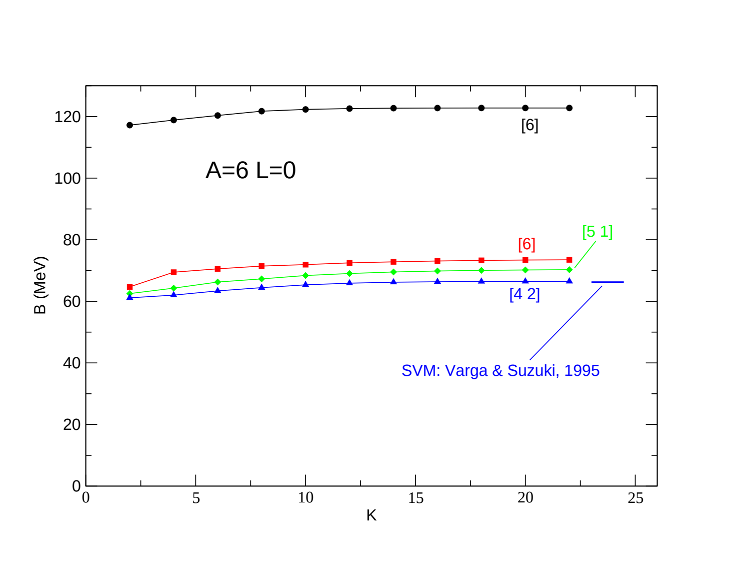
$\Psi = \sum_k a_k \widetilde{\Phi}_k$

Eigenvalue	[(2)]	irrep
− 7.9771E + 00	5.0000E + 00	[4, 1]
−7.9771E + 00	5.0000E + 00	
−7.9771E + 00	5.0000E + 00	
−7.9771E + 00	5.0000E + 00	
−5.0673E + 00	1.0000E + 01	[5]
−1.4406E − 01	5.0000E + 00	[4, 1]
−1.4406E − 01	5.0000E + 00	
−1.4406E − 01	5.0000E + 00	
−1.4406E − 01	5.0000E + 00	
1.9439E + 00	2.0000E + 00	[3, 2]
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
6.4904E + 00	−2.0000E + 00	[2, 2, 1]
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	

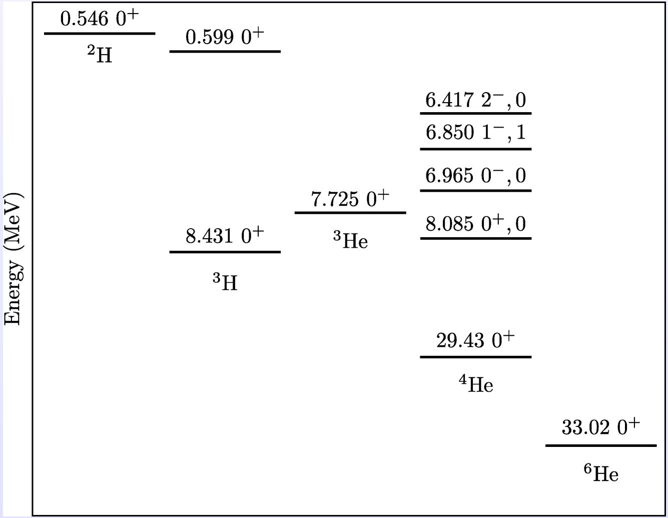
$A = 5$


A=5 L=1
HH & SVM
[4 1]
HH: Barnea, 1999
SVM: Varga & Suzuki, 1995
B (MeV)
K
44
43
42
41
40
39
0
5
10
15
20
25
30

$A = 6$ (


A=6 L=0
[6]
[6]
[5 1]
[4 2]
SVM: Varga & Suzuki, 1995
B (MeV)
K
0
20
40
60
80
100
120
0
5
10
15
20
25

Spectrum with the S-wave projecting Volkov potential



Conclusion of the second part

1. the method has been proved to work (arXiv:1009.3426)
   - HH: analytical properties, asymptotic behaviour
   - NCSM: second quantization

$$\sum_{i=1}^{A}\left[\frac{\boldsymbol{p}_i^2}{2m}+\frac{1}{2}m\omega\boldsymbol{r}_i^2\right]=\sum_{i=1}^{A}\left[\frac{\boldsymbol{p}_i^2}{2m}\right]+\frac{1}{2}m\omega'\rho^2$$

2. Work in progress:
   - inclusion of spin/isospin $\rightarrow$ “realistic” NN potentials
   - 3N interaction
   - scattering using the KVP (integral relations: arXiv:1002.3756)

Additional material

Transformation coefficients (2)

Hyperangles from $\xi_{i_1}, \xi_{i_2}, \xi_{i_3}, \xi_{i_4}$

$$\cos\phi_4 = \frac{\xi_{i_4}}{\sqrt{\xi_{i_1}^2 + \xi_{i_2}^2 + \xi_{i_3}^2 + \xi_{i_4}^2}}$$

$$\cos\phi_3 = \frac{\xi_{i_3}}{\sqrt{\xi_{i_1}^2 + \xi_{i_2}^2 + \xi_{i_3}^2}}$$

$$\cos\phi_2 = \frac{\xi_{i_2}}{\sqrt{\xi_{i_1}^2 + \xi_{i_2}^2}}$$

Note that when the change involves ξ_{i_1} and ξ_{i_2}, only ϕ_2 would change

A better strategy:

1. transform to a basis constructed in terms of $(\xi_2, \xi_3, \xi_1, \xi_4)$
   - $\mathcal{T}$ coefficients Kildyushov, SJNP **15**, 113 (1972)
2. transformation to HH functions constructed in terms of $(\xi_2', \xi_3', \xi_1, \xi_4)$
   - $\mathcal{R}$ coefficients Raynal-Revai, Nuov. Cim. A**68**, 612 (1970) (= transformation coefficients for $A = 3$)
3. return to transformation to the order $(\xi_1, \xi_2', \xi_3', \xi_4)$
   - $\mathcal{T}$ coefficients

Transformation coefficients (3)

Summary

- $\mathcal{C}^{ij,LM}_{[K][K']}$ =coefficients to transform the HH function so that $\boldsymbol{\xi}_N = \boldsymbol{r}_{ij}$

$$\mathcal{C}^{ij,LM}_{[K][K']} = \left[\mathcal{A}^{LM}_{i_1} \cdots \mathcal{A}^{LM}_{i_n}\right]_{[K][K']} .$$

- $[\mathcal{A}^{LM}_i]_{[K][K']}$ = coefficients coming from a transposition

$$[\mathcal{A}^{LM}_i]_{[K][K']} = \delta_{l_1,l'_1} \cdots \delta_{l_N,l'_N} \cdots {}^{(i)}\mathcal{A}^{L_{i-1}K_{i-1},L_{i+1}K_{i+1}}_{l_i,l'_i,l_{i+1},l'_{i+1},L_iK_i,L'_iK'_i}$$

- which can be written as a sum of products of $\mathcal{T}$ and $\mathcal{R}$ coefficients

$$[\mathcal{A}^{LM}_i]_{[K][K']} \sim \sum \mathcal{T}^{\alpha_{K_{i-1}}\,\alpha_{l_i}\,\alpha_{l_{i+1}}}_{n_i\tilde{n}_iK_{i+1}} \mathcal{T}^{\alpha_{K_{i-1}}\,\alpha_{l'_i}\,\alpha_{l'_{i+1}}}_{n'_i\tilde{n}_iK_{i+1}} \mathcal{R}^{\tilde{K}_i,\tilde{L}_i}_{l_{i+1}l_i,l'_{i+1}l'_i} ,$$

Publications 2009/2010

1. Electromagnetic currents and magnetic moments in chiral effective field theory (χEFT), S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, Phys. Rev. C **80**, 034004 (2009)
2. Relativity constraint on the two-nucleon contact interaction, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, `arXiv:1001:3676`
3. Comparative studies of three-nucleon force models in $A = 3, 4$ systems, A. Kievsky *et al.* `arXiv:1002.1601`
4. Variational description of continuum states in terms of integral equations, A. Kievsky *et al.* `arXiv:1002.3756`
5. The parity-violating asymmetry in the ${}^3\mathrm{He}(\vec{n}, p){}^3\mathrm{H}$ reaction, M. Viviani *et al.* `arXiv:1007.2052`
6. Muon capture on deuteron and ${}^3\mathrm{He}$, L. E. Marcucci *et al.* `arXiv:1008.0356`