Scaling in light nuclei with a weakly bound two-neutron halo

Tobias Frederico
Instituto Tecnológico de Aeronáutica
São José dos Campos – Brazil
tobias@ita.br

❖ Collaborators

- Mohammadreza Hadizadeh – IFT-Unesp/São Paulo
- Marcelo T. Yamashita – IFT-Unesp/São Paulo
- Raquel S. Marques de Carvalho – UNIFESP/São Paulo
- Lauro Tomio – UFF/Niterói & IFT-Unesp/São Paulo
- Antonio Delfino – UFF/Niterói

OUTLINE

Thomas-Efimov Physics of weakly bound three-body systems

Two neutron weakly bound three-body halo nuclei

Scales of s-wave n-n-c system: contact interaction

Classification scheme, sizes and nn-correlation

Threshold conditions for excited Efimov states

^{20}C structure and n-^{19}C scattering

^{22}C structure

Four-body scale

Summary and outlook

Physics of weakly bound three-body systems

Charateristic phenomena three-body systems:

Thomas collapse (1935) and Efimov effect (1970)

$r_o \rightarrow 0$ $|a| \rightarrow \infty$

??? infinitely many weakly bound states

$|a|/r_o \rightarrow \infty$

Thomas-Efimov effect!

S.K. Adhikari, A. Delfino, T. Frederico, I.D. Goldman, and L. Tomio, Phys. Rev. A **37**, 3666 (1988).

One three-body scale is necessary to represent short-range physics !!!!

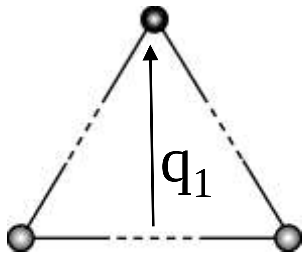
A. S. Jensen, K. Riisager, D. V. Fedorov, and E. Garrido, Rev. Mod. Phys. **76**, 215 (2004).

E. Braaten, H.-W. Hammer, Phys. Rep. **428**, 259 (2006)

Weakly bound system wave function & contact interaction

Three-boson wave function:

$$(E - H_0)\psi = 0$$

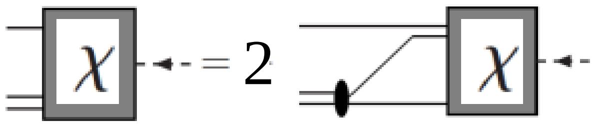


$$\psi = \int d^3 q_1 \frac{\exp\{i[E - (3/4)q_1^2]^{1/2} R_1\}}{R_1} e^{i\mathbf{q}_1\cdot\mathbf{r}_1} \chi(\mathbf{q}_1)$$

$$+ \quad (1 \rightarrow 2) \;+\; (1 \rightarrow 3)$$

Zero-range 3-boson equation: Thomas-Efimov effect

Skorniakov and Ter-Martirosian equations (1956)



$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm\sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}\vec{y}^2}} \int d^3x \left(\frac{1}{\epsilon_3 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} - \frac{1}{1 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} \right) \chi(\vec{x})$$

$\epsilon_3 = E_3 / \mu_{(3)}^2$ $\quad \epsilon_2 = E_2 / \mu_{(3)}^2$ $\quad \mu_{(3)}^2 = 1$

Adhikari,TF,Goldman, PRL74 (1995) 487

Thomas collapse: $\mu_{(3)}^2 \rightarrow \infty$

$\epsilon_2 = E_2 / \mu_{(3)}^2$

Efimov effect: $E_2 \rightarrow 0$

Scale invariance at the unitary limit

$$\epsilon_3 = \epsilon_2 = 0$$

s-wave: $$\chi(y) = \frac{4}{\pi\sqrt{3}y}\int_0^\infty dx\, x^2\,\chi(x)\int_{-1}^{1} dz\,\frac{1}{x^2+y^2+x\,y\,z}$$

Solution: $$\chi(y) = y^{s-2}$$

Efimov equation: $$1 = \frac{8}{\sqrt{3}s}\frac{\sin(\pi s/6)}{\cos(\pi s/2)} \qquad s = \pm i s_0 \qquad s_0 \approx 1.00624$$

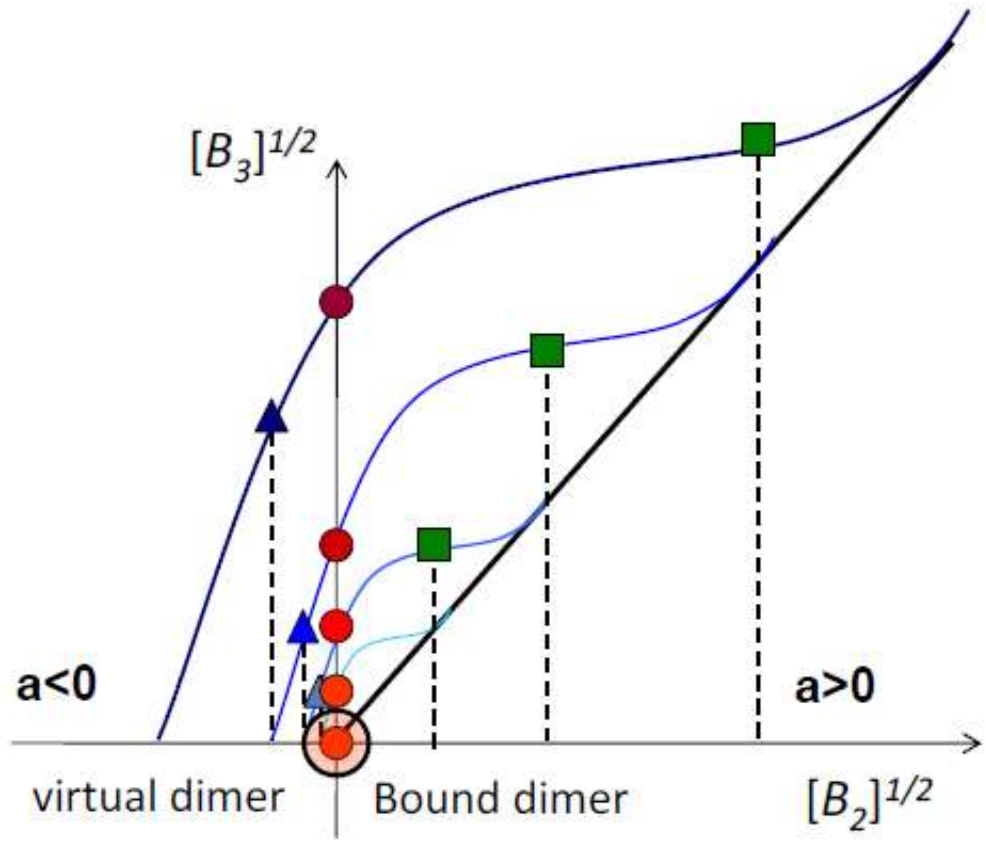
$$\chi(y) = a_+ y^{i s_0 - 2} + a_- y^{-i s_0 - 2}$$

G.S. Danilov, Sov. Phys. JETP 13 (1961) 349

$$\chi(y) = y^{-2}\sin\left(s_0 \ln y + c\right)$$

One *parameter* to fix the solution → one *3-body scale*!

Efimov Plot



Scaling limit & limit cycle

$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)}(\pm\sqrt{\epsilon_2})$$

$$\xi \equiv \pm\sqrt{\epsilon_2} = \pm(E_2\epsilon_3^{(N)}/E_3^{(N)})^{1/2}$$

$$\frac{E_3^{(N+1)}}{E_3^{(N)}} = \lim_{N\to\infty} \frac{\epsilon_3^{(N+1)}(\xi)}{\epsilon_3^{(N)}} = \mathcal{F}\left(\pm\sqrt{\frac{E_2}{E_3^{(N)}}}\right)$$

Scaling function

$$\mathcal{F}(0) = e^{2\pi/s_0} = 1/515$$

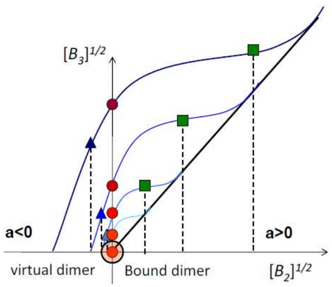
Efimov 1970

Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

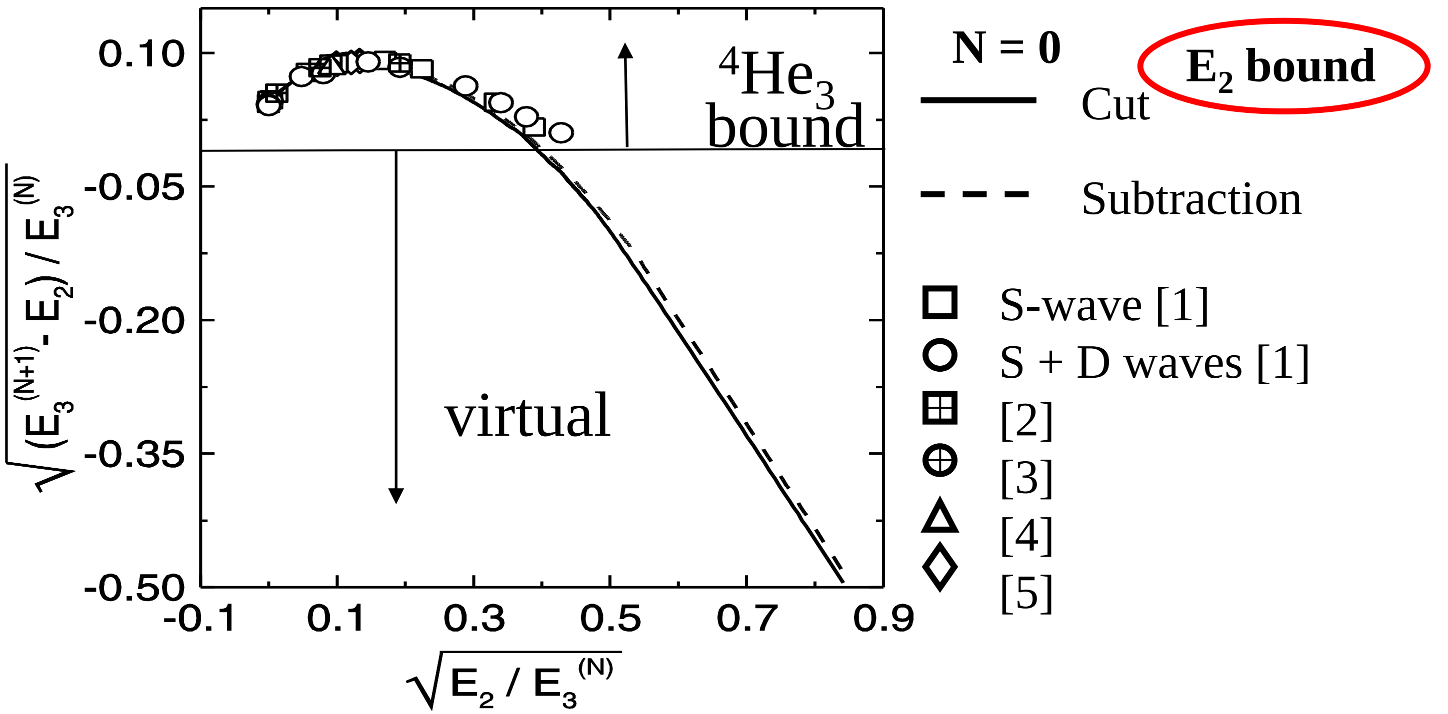
Limit cycle:
Mohr et al Ann.Phys. 321 (2006)225

Efimov States – Bound and virtual states (3 identical bosons)

T. Frederico, LT, A. Delfino and E. A. Amorim. *Phys. Rev.* **A60**, R9 (1999).



Scaling plot



[1] Th. Cornelius e W. Glöckle. *J. Chem. Phys.* **85**, 1 (1996).
[2] S. Huber. *Phys. Rev.* **A31**, 3981 (1985).
[3] P. Barletta e A. Kievsky. *Phys. Rev.* **A64**, 042514 (2001).
[4] D. V. Fedorov e A. S. Jensen. *J. Phys.* **A34**, 6003 (2001).
[5] E. A. Kolganova, A. K. Motovilov e S. A. Sofianos. *Phys. Rev.* **A56**, R1686 (1997).

Range correction: Thogersen, Fedorov, Jensen PRA78(2008)020501(R)

Scaling functions: Correlation between observables

$$O(E, E_3, E_2) = (E_3)^{\eta} \mathcal{A}(\sqrt{E/E_3}, \sqrt{E_2/E_3})$$

Scaling function

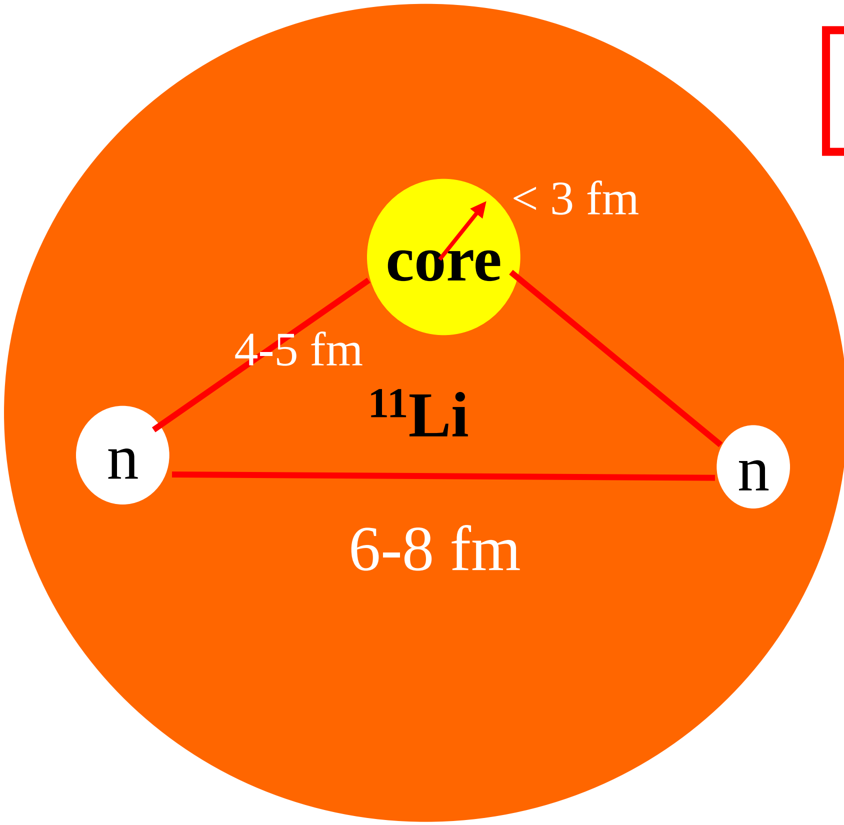
Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

Limit cycle:
Mohr et al Ann.Phys. 321 (2006)225

Correlation between S-wave observables

- Phillips plot: triton B.E. versus doublet scattering length
- 2nd order n-d polarization observables versus triton B.E.
- Trapped atomic trimer B.E. versus recombination rate

Two neutron weakly bound three-body halo nuclei



Tanihata et al., PRL**55**, 2676 (1985)

core-neutron-neutron halo nuclei

^{11}Li ^{14}Be ^{20}C ^{22}C

Binding energy ~ MeV or < MeV

R_{nn}(Exp) ~ 6 - 8 fm (^{11}Li)

F. M. Marqués et al. Phys. Rev. C **64**, 061301 (2001)

M. Petrascu et al. Nucl. Phys. A **738**, 503 (2004)

^{20}C ^{22}C K. Tanaka *et al.*, Phys. Rev. Lett. **104** (2010) 062701

Scales of s-wave n-n-c system: contact interaction

E_{nn} Energy of the bound/virtual nn system

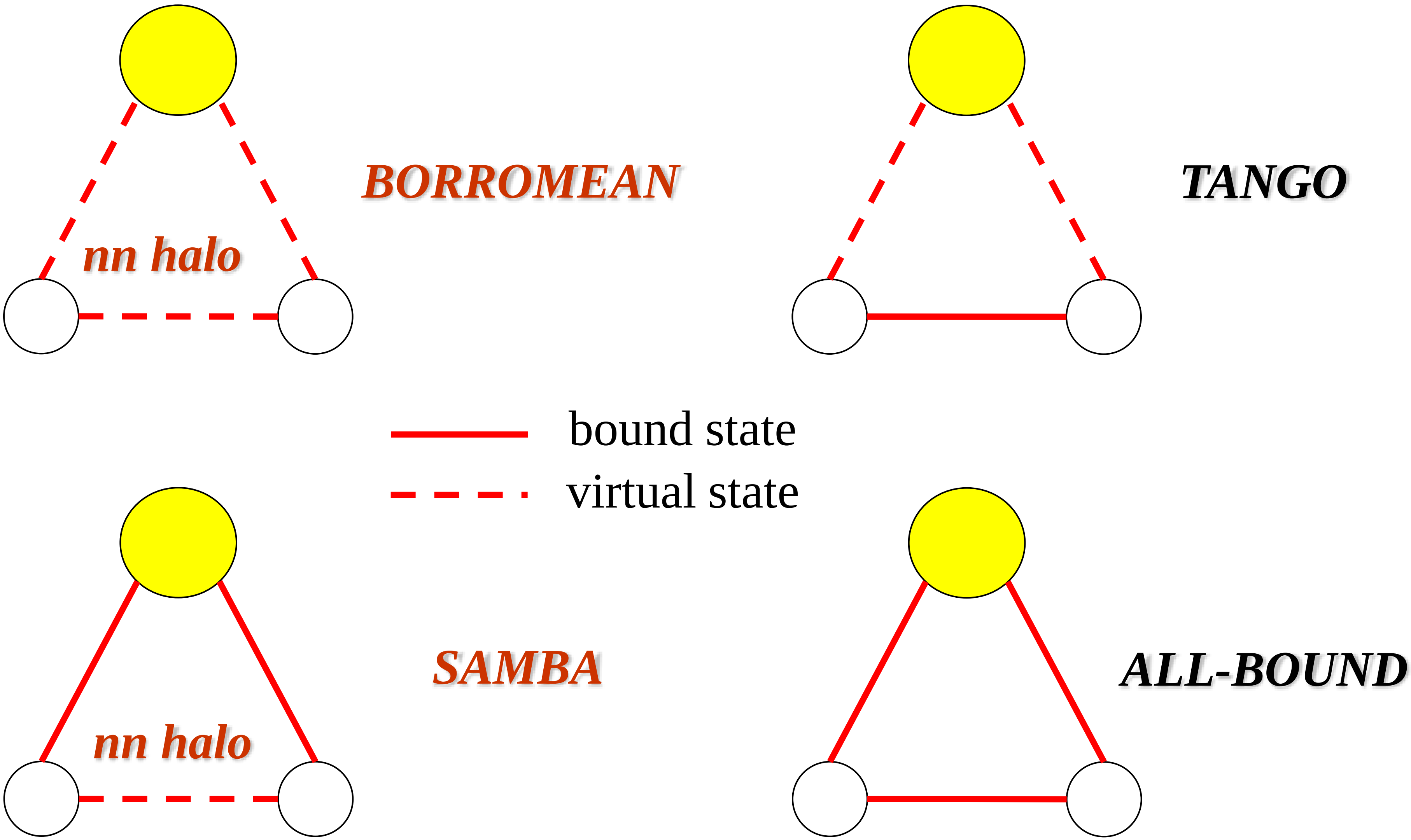
E_{nc} Energy of the bound/virtual nc system

$B_N = |E_3^{(N)}|$ Energy of the Nth state of the nnc system

A = mass of the core

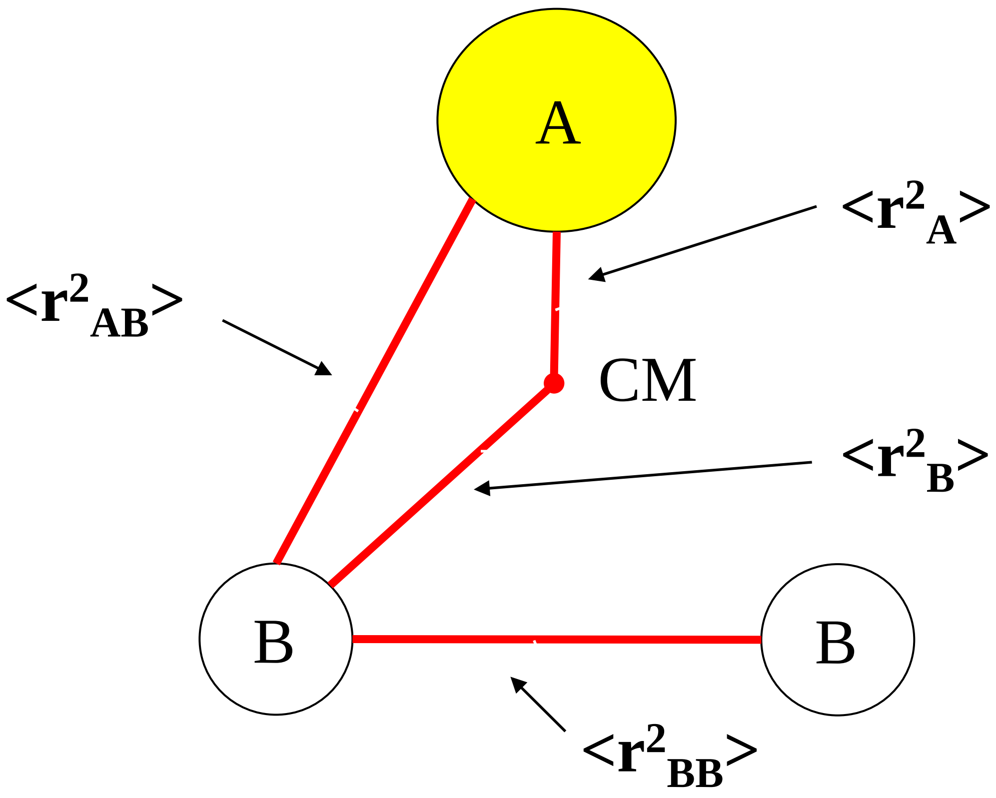
Classification scheme

Yamashita, Tomio and T. F. Nucl. Phys. A 735, 40 (2004)



A.S. Jensen, K. Riisager, D.V. Fedorov, E. Garrido, Europhys. Lett. 61 (2003) 320.
F. Robicheaux, Phys. Rev. A 60 (1999) 1706.

Root mean square radii



Root mean square radii

Scaling functions for the radii

$$\sqrt{\left\langle r_{A\gamma}^{2}\right\rangle \left|E_3\right|} = R_{A\gamma}\left(\pm\sqrt{\frac{E_{AA}}{E_3}}, \pm\sqrt{\frac{E_{AB}}{E_3}}, \mathrm{A}\right)$$

$$\sqrt{\left\langle r_{\gamma}^{2}\right\rangle \left|E_3\right|} = R_{\gamma}^{CM}\left(\pm\sqrt{\frac{E_{AA}}{E_3}}, \pm\sqrt{\frac{E_{AB}}{E_3}}, \mathrm{A}\right)$$

$\gamma = A$ or B

+ two-body bound state
- two-body virtual state

Root mean square radii: Core+neutron+neutron

Exp: F.M. Marqués, et al., Phys. Lett. B 476 (2000) 219;

Core (A)	$-E_3$ (MeV)	$-E_{nA}$ (MeV)	$\sqrt{\langle r_{nn}^2 \rangle}$ (fm)	$\sqrt{\langle r_{nn}^2 \rangle_{exp}}$ (fm)
^{4}He	0.973	0 (v)	5.1	
		0.3 (v)	4.6	5.9±1.2
		4.0 [23] (v)	3.6	
^{9}Li	0.32	0 (v)	9.2	6.6±1.5
		0.8 [24] (v)	5.9	
^{9}Li	0.29	0 (v)	9.7	
		0.05 [20,25,26] (v)	8.5	6.6±1.5
		0.8 [24] (v)	6.7	
^{12}Be	1.337	0 (v)	4.6	5.4±1.0
		0.2[27] (v)	4.2	
^{18}C	3.50	0.16 [3]	3.0	–
		0.53 [14]	4.4	–

Yamashita, Tomio and T. F.
NPA 735, 40 (2004)

Nucleus	B_3 [keV]	E_{nc} [keV]	r_0 [fm]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]	$\sqrt{\langle r_n^2 \rangle}$ [fm]	$\sqrt{\langle r_c^2 \rangle}$ [fm]
^{11}Li	247	−25	0.0	8.7 ± 0.7	7.1 ± 0.5	6.5 ± 0.5	1.0 ± 0.1
	247	−25	1.4	8.80 ± 0.07	7.21 ± 0.06	6.51 ± 0.05	1.040 ± 0.008
	247	−800 [48]	0.0	6.8 ± 1.8	5.9 ± 1.5	5.3 ± 1.4	0.9 ± 0.2
	247	−800 [48]	1.4	6.3 ± 0.5	5.5 ± 0.4	4.9 ± 0.4	0.81 ± 0.06
^{14}Be	1120	−200 [49]	0.0	4.1 ± 0.5	3.5 ± 0.5	3.2 ± 0.4	0.40 ± 0.05
	1120	−200 [49]	1.4	3.86 ± 0.09	3.29 ± 0.08	3.02 ± 0.07	0.384 ± 0.009
^{12}Be	3673	503	0.0	3.0 ± 0.6	2.5 ± 0.5	2.3 ± 0.5	0.32 ± 0.07
	3673	503	1.4	3.3 ± 0.2	2.7 ± 0.1	2.5 ± 0.1	0.35 ± 0.02
^{18}C	4940	731	0.0	2.6 ± 0.7	2.2 ± 0.6	2.1 ± 0.5	0.18 ± 0.05
	4940	731	1.4	2.9 ± 0.2	2.4 ± 0.2	2.3 ± 0.2	0.21 ± 0.01
^{20}C	3506	530 [45]	0.0	3.0 ± 0.7	2.5 ± 0.6	2.4 ± 0.5	0.19 ± 0.04
	3506	530 [45]	1.4	3.38 ± 0.18	2.75 ± 0.15	2.60 ± 0.14	0.21 ± 0.01
	3506	162	0.0	2.8 ± 0.3	2.4 ± 0.3	2.3 ± 0.3	0.19 ± 0.02
	3506	162	1.4	3.03 ± 0.06	2.53 ± 0.05	2.39 ± 0.05	0.198 ± 0.004
	3506	60	0.0	2.8 ± 0.2	2.3 ± 0.2	2.2 ± 0.2	0.18 ± 0.01
	3506	60	1.4	2.84 ± 0.03	2.41 ± 0.03	2.28 ± 0.03	0.192 ± 0.002
^{20}C*	65.0 ± 6.8	60	0.0	42 ± 3	38 ± 3	41 ± 3	2.2 ± 0.2
^{20}C*	64.9 ± 0.7	60	1.4	43.2 ± 0.5	38.7 ± 0.4	42.9 ± 0.5	2.26 ± 0.02

Canham and Hammer
NPA 836 (2010) 275

More on n-n-c Samba Nuclei…

$$R \sim \sqrt{\frac{2}{A}\langle r^2_{n-\mathrm{CM}}\rangle + \frac{A-2}{A}R^2_{\mathrm{core}}}$$

$$R_{\mathrm{core}} = 1.013(A-2)^{1/3}$$

experimental radius of ^{12}C of $R = 2.32$ fm

dipole strength

$$B(E1) = \frac{3e^2}{16\pi\mu}\left(\frac{2Z}{A}\right)^2 \frac{1}{E_{2n}}$$

Table 1. Physical quantities of the Samba halo nuclei given in the first column. The second and third columns are, respectively, the n–c and the n–n–c energies used to calculate the n-CM root-mean-square radii, $\sqrt{\langle r^2_{n-\mathrm{CM}}\rangle}$. R_0 is the nucleus radius and R_{core} is the core radius, R (Eq. (2)) is the average between R_0 and $\sqrt{\langle r^2_{n-\mathrm{CM}}\rangle}$. $B(E1, 0 \to 1)$ is given by Eq. (1). $T_{1/2}$ is the nucleus half-life.

Nucleus	E_n (MeV)	E_{2n} (MeV)	$\sqrt{\langle r^2_{n-\mathrm{CM}}\rangle}$ (fm)	R_0 (fm)	R_{core} (fm)	R (fm)	$B(E1, 0\to 1)$ ($e^2\mathrm{fm}^2$)	$T_{1/2}$
^{12}Be	0.504	3.669	4.81	2.75	2.18	2.80	0.043	21.3 ms
^{15}B	0.973	3.734	5.15	2.96	2.38	2.91	0.037	9.87 ms
^{20}C	0.191	3.462	4.00	3.26	2.66	2.82	0.013	14 ms
^{23}N	1.200	3.672	6.07	3.41	2.80	3.22	0.024	37.7 s
^{27}F	1.041	2.412	8.94	3.60	2.96	3.75	0.051	$>$ 200 ns

Estimatives from Yamashita, Frederico, Hussein, MPLA22(2006)1749

$$^{12}\mathrm{Be},\ R_{\mathrm{exp}} = 2.59 \pm 0.06\ \mathrm{fm}$$

Neutron-neutron correlation function

Radii are experimentally extracted from

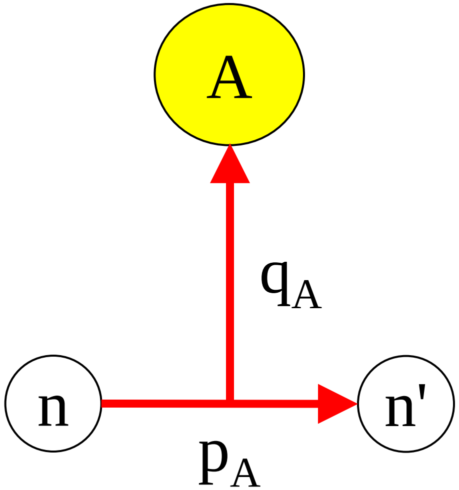
correlation function

R. Hanbury-Brown and R. Q. Twiss (HBT) - NATURE
177, 27 (1956)
178, 1046 (1956)
178, 1447 (1956)

First used in astrophysics

Nuclear Physics

Neutron-neutron correlation function



$$C_{nn}(\vec{p}_A) = \frac{\int d^3 q_A \left|\Phi(\vec{q}_A, \vec{p}_A)\right|^2}{\int d^3 q_A \rho(\vec{q}_n')\rho(\vec{q}_n)}$$

$$\vec{q}_{n'} = \vec{p}_A - \frac{\vec{q}_A}{2} \qquad \vec{q}_n = -\vec{p}_A - \frac{\vec{q}_A}{2}$$

One-body density

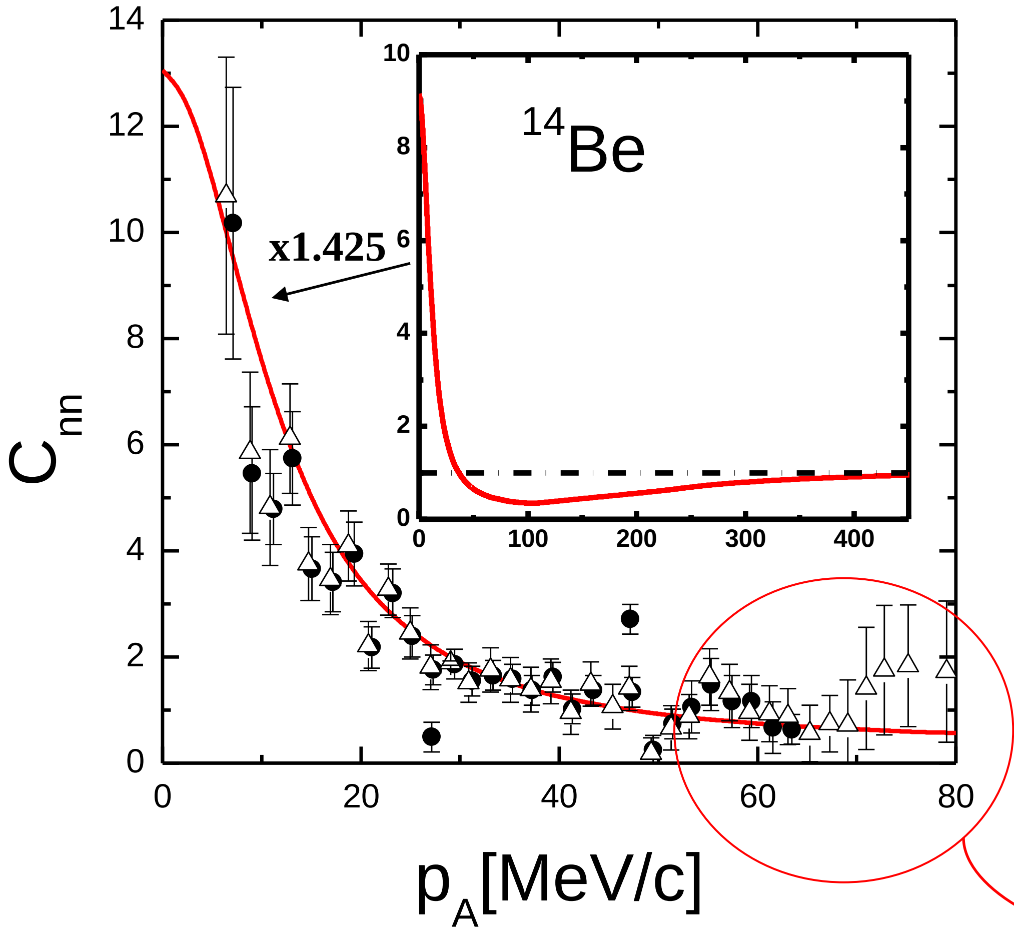
$$\rho(\vec{q}_{nA}) = \int d^3 q_{n'A} \left|\Phi\left(-\vec{q}_{nA} - \vec{q}_{n'A}, \frac{\vec{q}_{nA} - \vec{q}_{n'A}}{2}\right)\right|^2$$

$\Phi \equiv \Phi(\vec{q}_A, \vec{p}_A)$ Breakup amplitude including the FSI between the neutrons

$$\Phi = \Psi(\vec{q}_A, \vec{p}_A) + \frac{1/(2\pi^2)}{\sqrt{E_{nn}} - ip_A} \int d^3 p \frac{\Psi(\vec{q}_A, \vec{p})}{p_A^2 - p^2 + i\varepsilon}$$

Ψ is the three-body wave function

Neutron-neutron correlation function



● F. M. Marqués et al. Phys. Rev. C **64**, 061301 (2001)

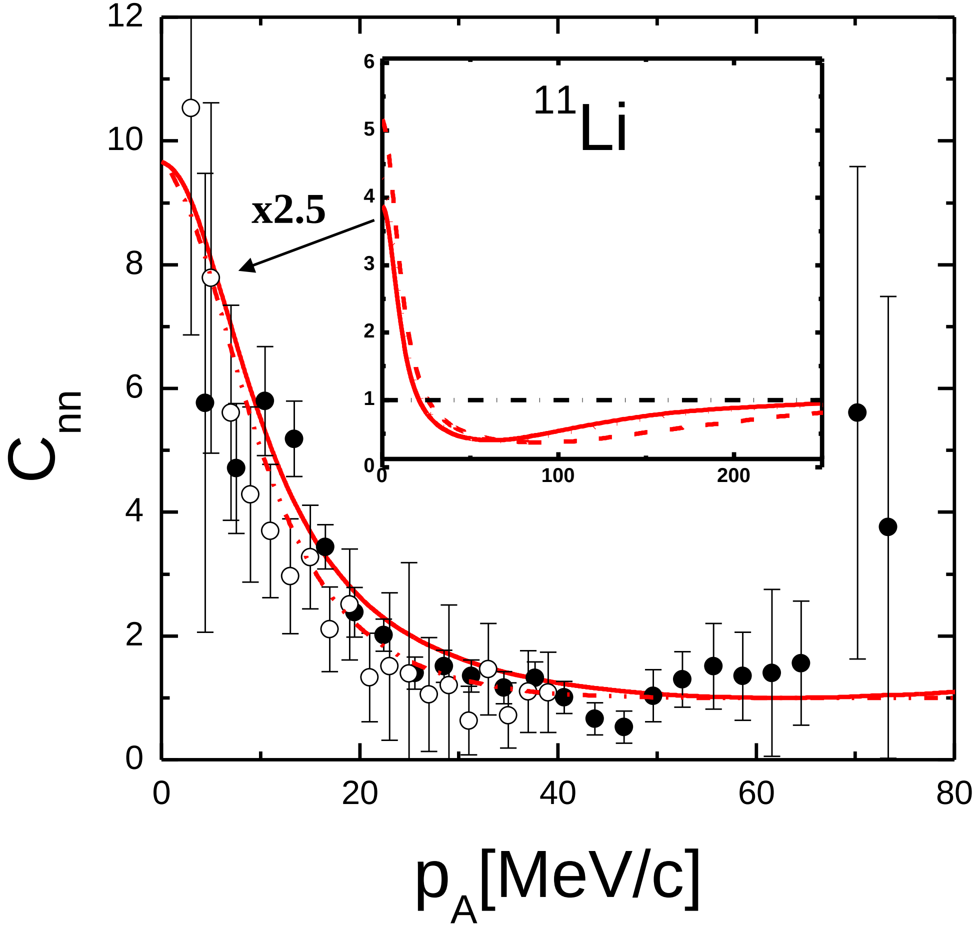
△ F. M. Marqués et al. Phys. Lett. B **476**, 219 (2000)

$E_3 = 1.337$ MeV
$E_{nA} = 0.2$ MeV
$E_{nn} = 0.143$ MeV

asymptotic region ?

M. T. Yamashita, T. Frederico and L. Tomio Phys. Rev. C 72, 011601(R) (2005)

Neutron-neutron correlation function



●

F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)

○ — · · —

M. Petrascu et al.
Nucl. Phys. A **738**, 503 (2004)

—— E_3 = 0.29 MeV
E_{nA} = 0.05 MeV

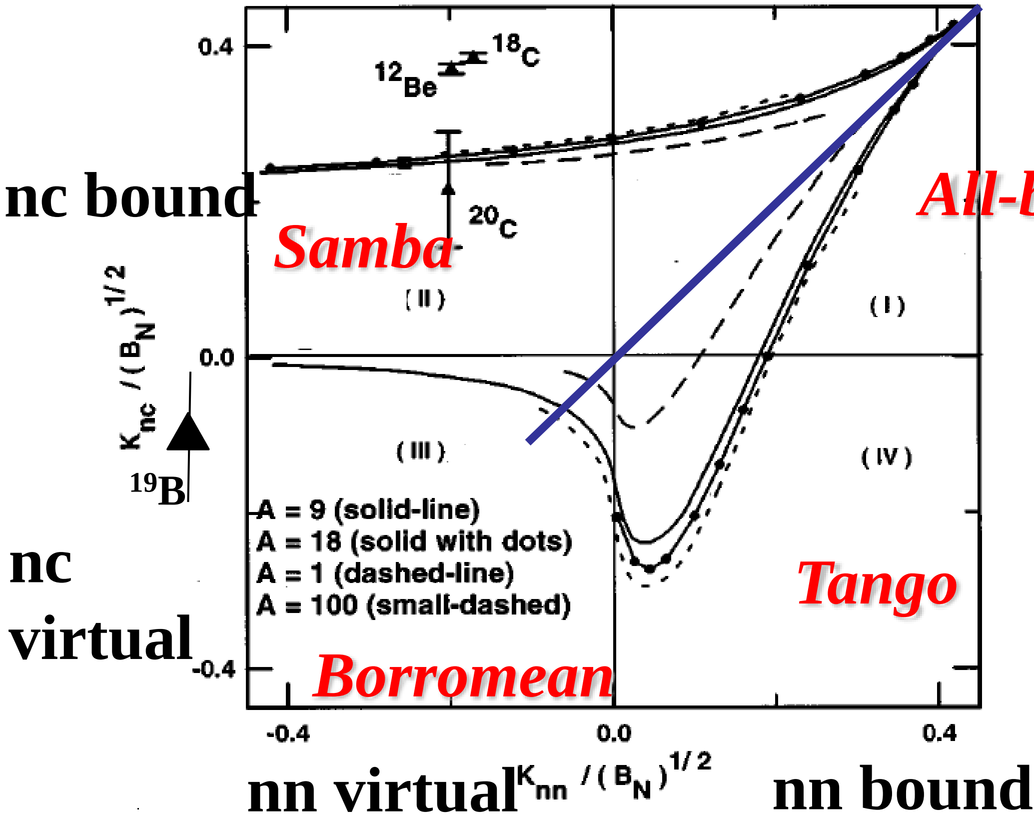
- - - E_3 = 0.37 MeV
E_{nA} = 0.8 MeV

······ E_3 = 0.37 MeV
E_{nA} = 0.05 MeV

Enn = 0.143 MeV

Threshold for an excited Efimov state: Halo-nuclei

Critical condition for an excited (N+1)-th Efimov state above the N-th one:



$K_{nn}=(B_{nn})^{1/2}$

$K_{nc}=(B_{nc})^{1/2}$

Amorim,TF,Tomio PRC56(1997)2378

Canham and Hammer EPJ A 37 (2008) 367; NPA 836 (2010) 275

"First Evidence for a Virtual ^{18}B Ground State" A. Spyrou et al Phys. Lett. B **683** (2010) 129

▲ ^{19}B $E_v(^{17}B) < -9$ KeV $S_{2n}(^{19}B) \sim 500 \pm 400$ KeV

Threshold for an excited Efimov state: molecules

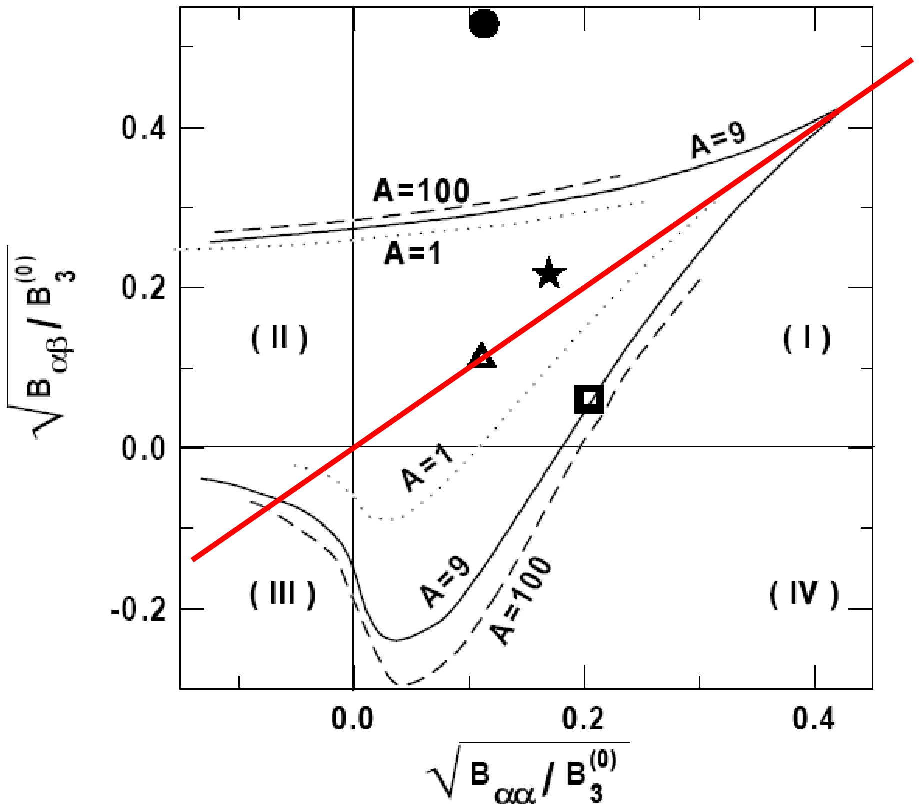


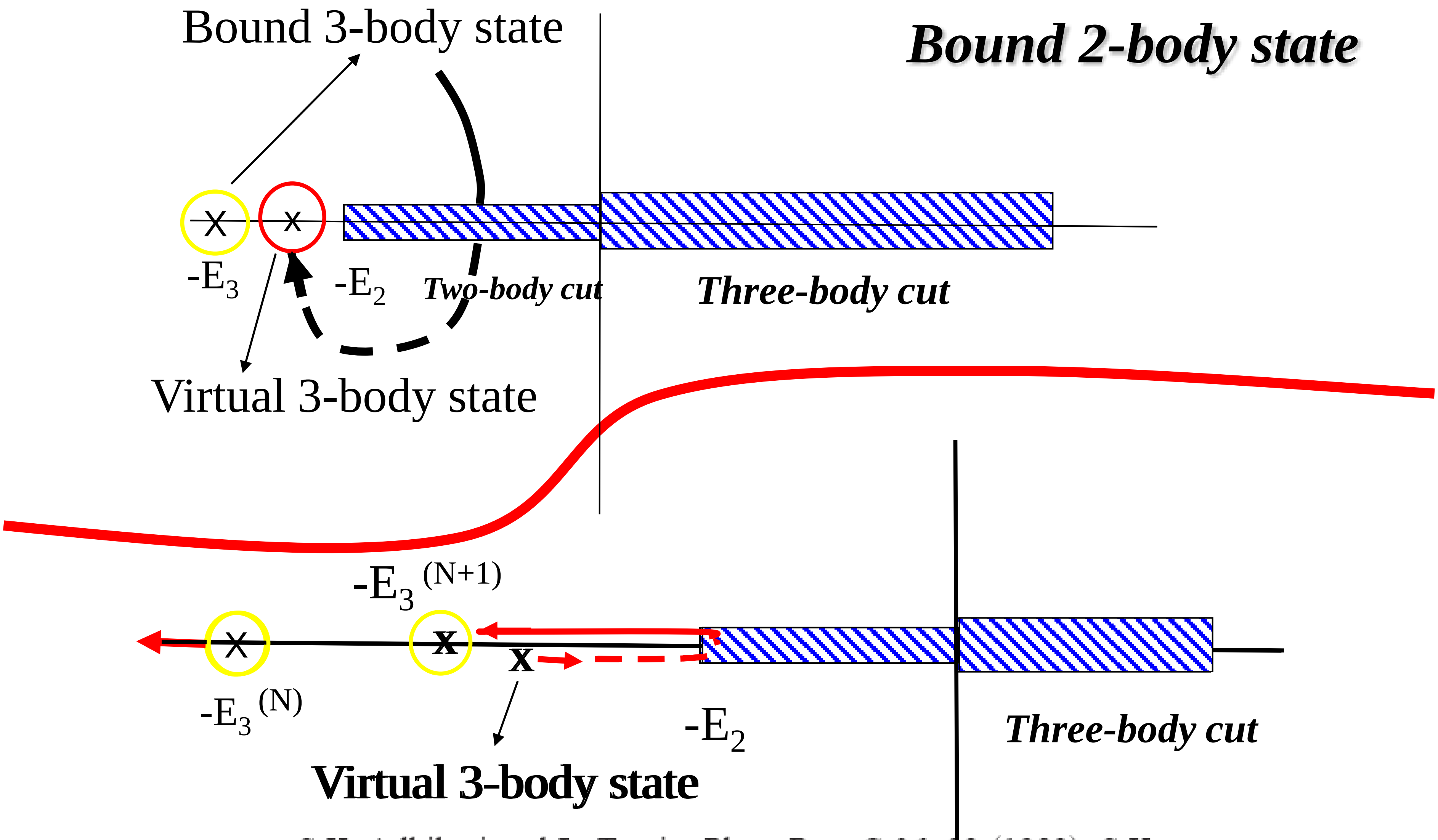
FIG. 1. The scaling approach for the three-body system $\alpha - \alpha - \beta$. The coordinates of the systems with $\alpha \equiv {}^4$He and $\beta = {}^4$He, ^{7}Li, ^{6}Li and ^{23}Na are respectively represented by a triangle, a star, a square and a full circle.

$K_{aa}=(B_{aa})^{1/2}$

$K_{ab}=(B_{ab})^{1/2}$

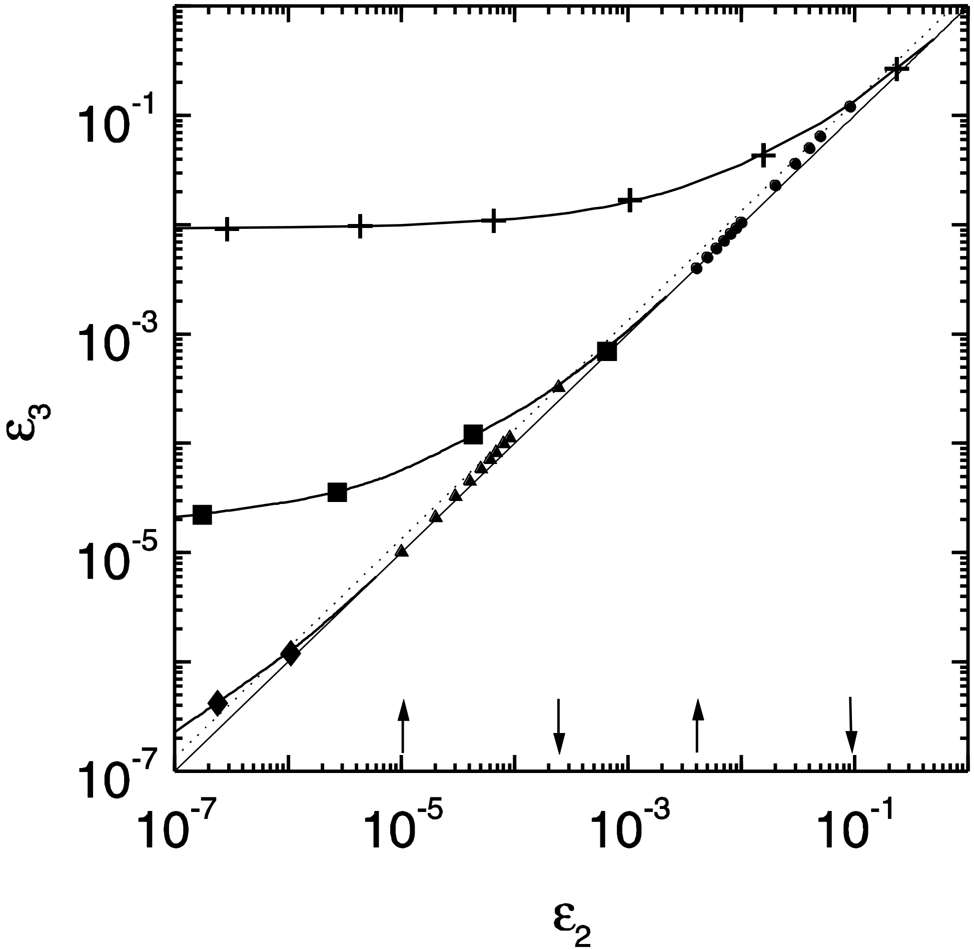
Delfino et al., JCP 113 (2000) 7874

Three-bosons: analytic structure & Efimov state trajectory



S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).

Efimov States – Bound and virtual states (3 identical bosons)



Bound-states (lines with simbols)

With Plus - Fundamental state

With squares – 1st. excited

With diamonds – 2nd. excited

Virtual-states (just simbols)

circles – 1st. Excited state

triangles – 2nd. Excited state

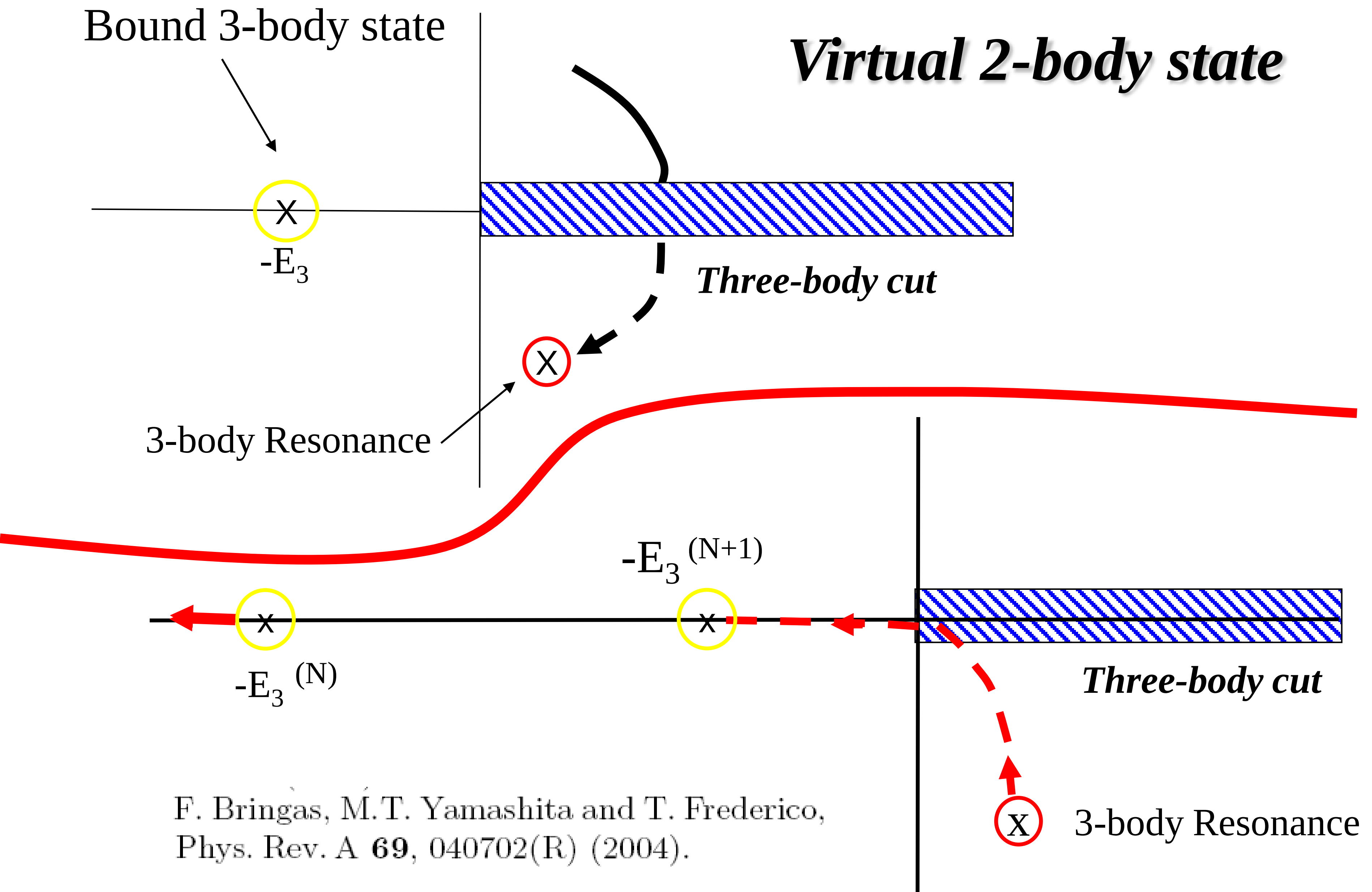
ε_2 bound

↓ the virtual state starts (dotted line) $\varepsilon_3 = \frac{4}{3}\varepsilon_2$

↑ Virtual-state turns to an excited state (continuous line) $\varepsilon_3 = \varepsilon_2$

S. K. Adhikari, A. C. Fonseca and LT *Phys. Rev.* **C27**, 1826 (1983).

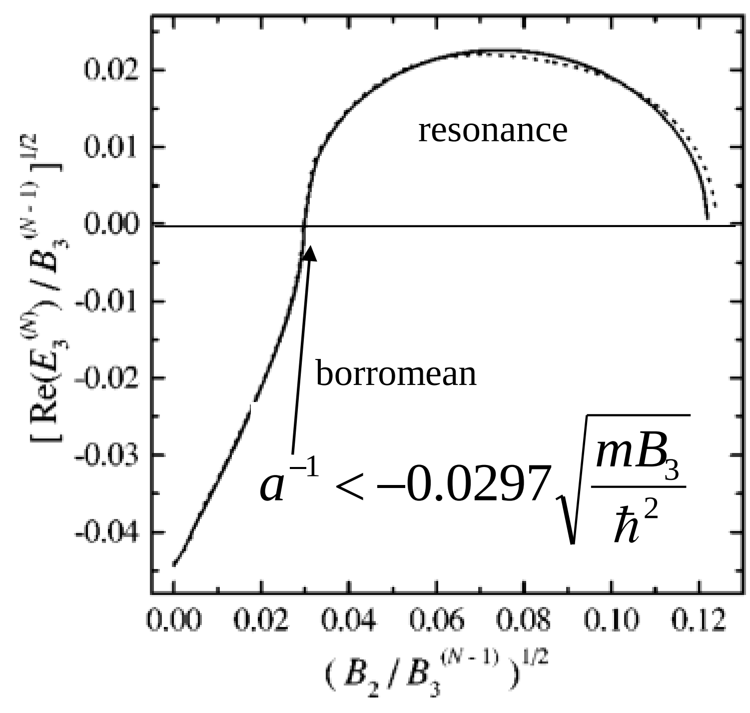
Borromean configuration: analytic structure & Efimov state trajectory

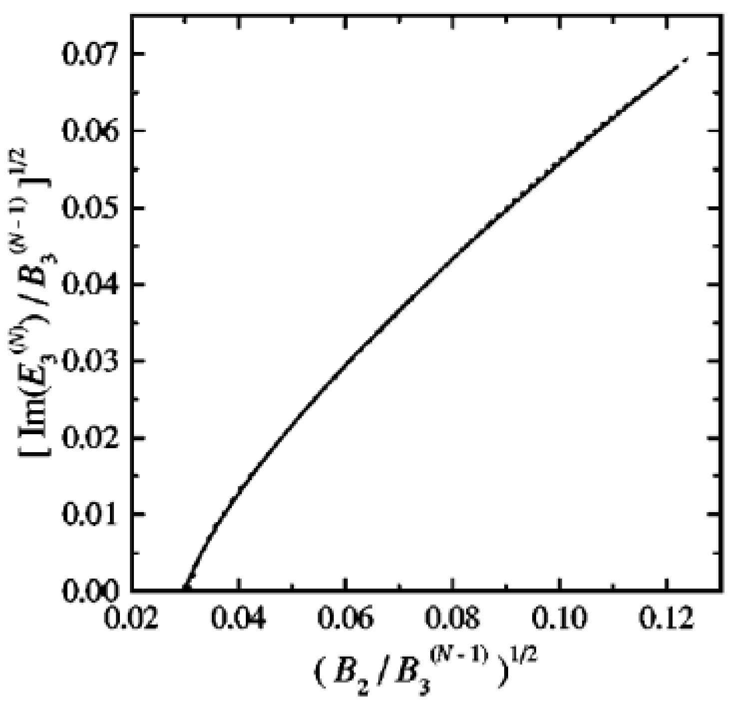


F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

Efimov state trajectory: borromean case

S-wave three-boson resonance





F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

Evidence of continuum resonances in recombination of ultracold Cs atoms

T. Kraemer et al, Nature **440**, 315 (2006)

Evidence of continuum resonances in ultracold cesium gas

T. Kraemer et al, Nature **440**, 315 (2006)

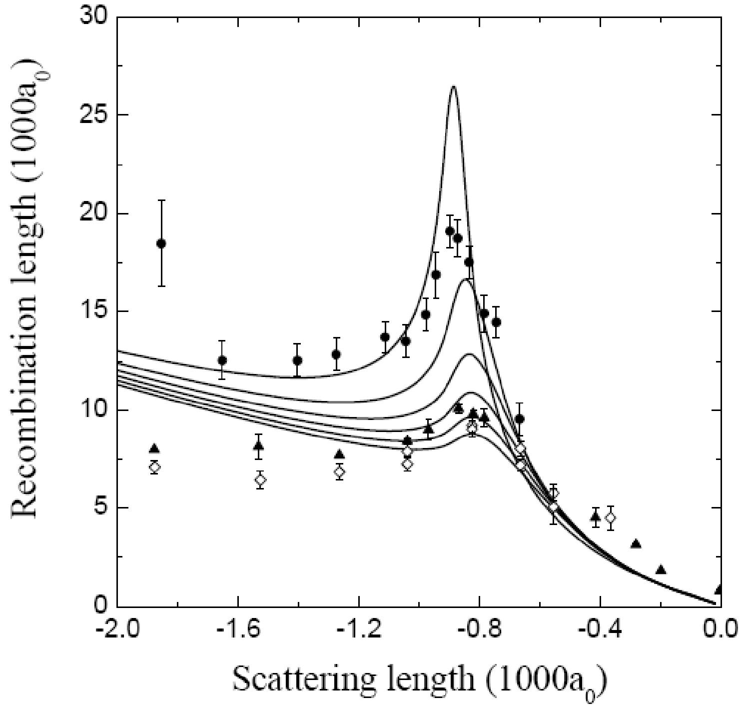
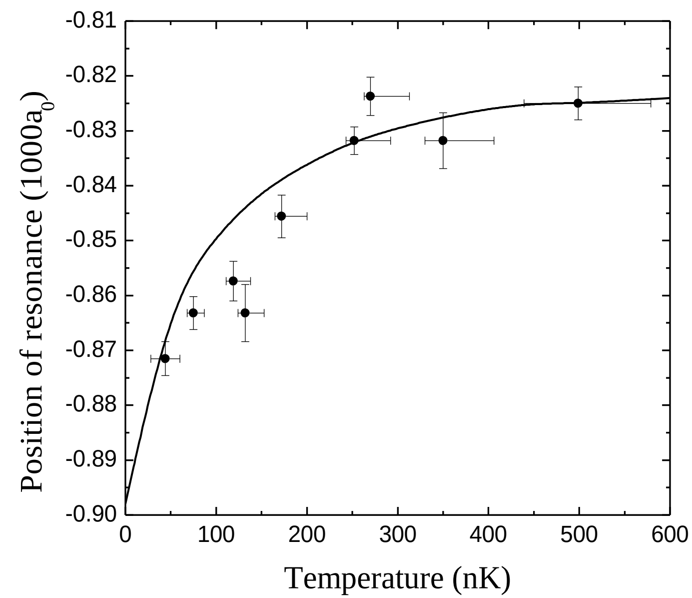


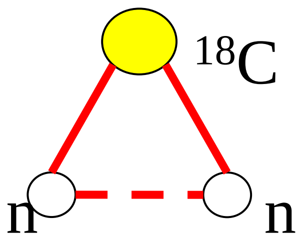
FIG. 2: Recombination length ($\rho_3 = [2m/(\sqrt{3}\hbar)\langle L_3\rangle_T]$) in the cesium trapped gas as a function of the scattering length and temperature. The solid curves from up to bottom are the theoretical results for T = 10 nK, 100 nK, 200 nK, 300 nK, 400 nK and 500 nK. The symbols are the experimental results for T = 10 nK (full circles), 200 nK (full triangles) and 250 nK (open diamonds) from Ref. [4].

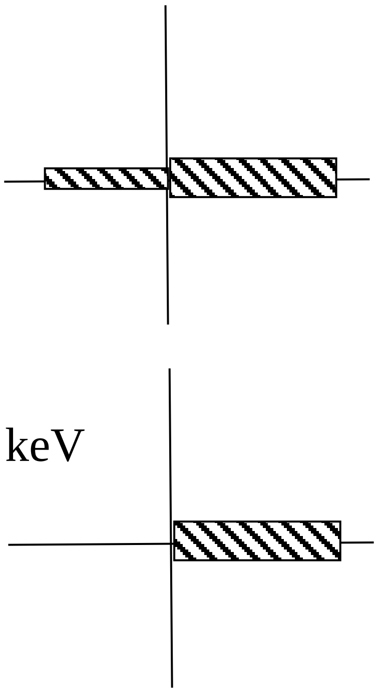
Position of the maximum of the recombination length as a function of the temperature. Experimental data from B. Engeser et al., *in preparation*.



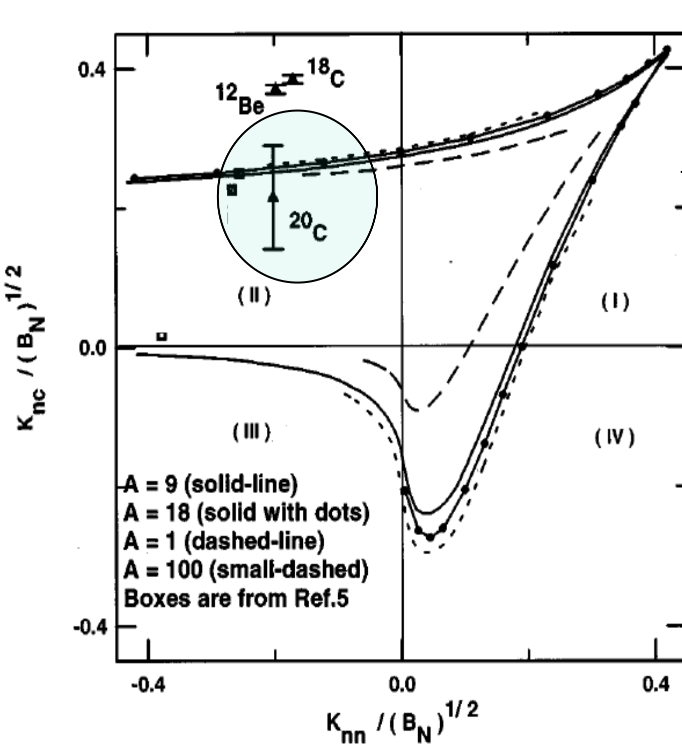
M.T. Yamashita et al. / Physics Letters A 363 (2007) 468

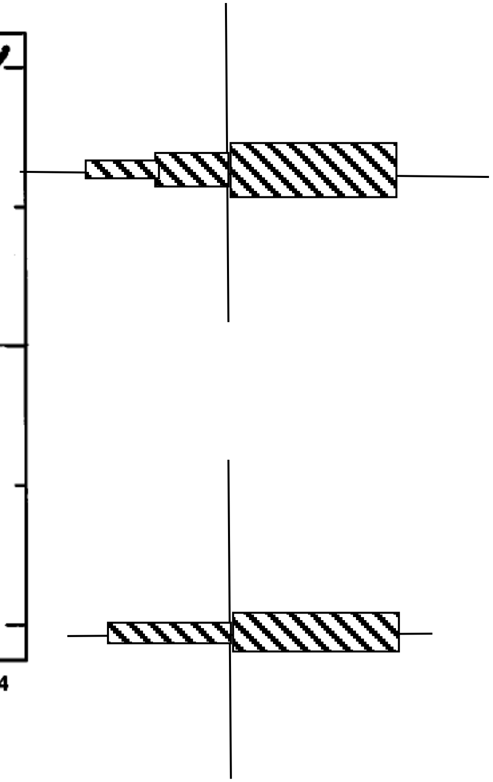
Threshold for an excited Efimov state and trajectory: ^{20}C





$E_3 = 3.5$ MeV
$E_{nc} = 160 \pm 110$ keV





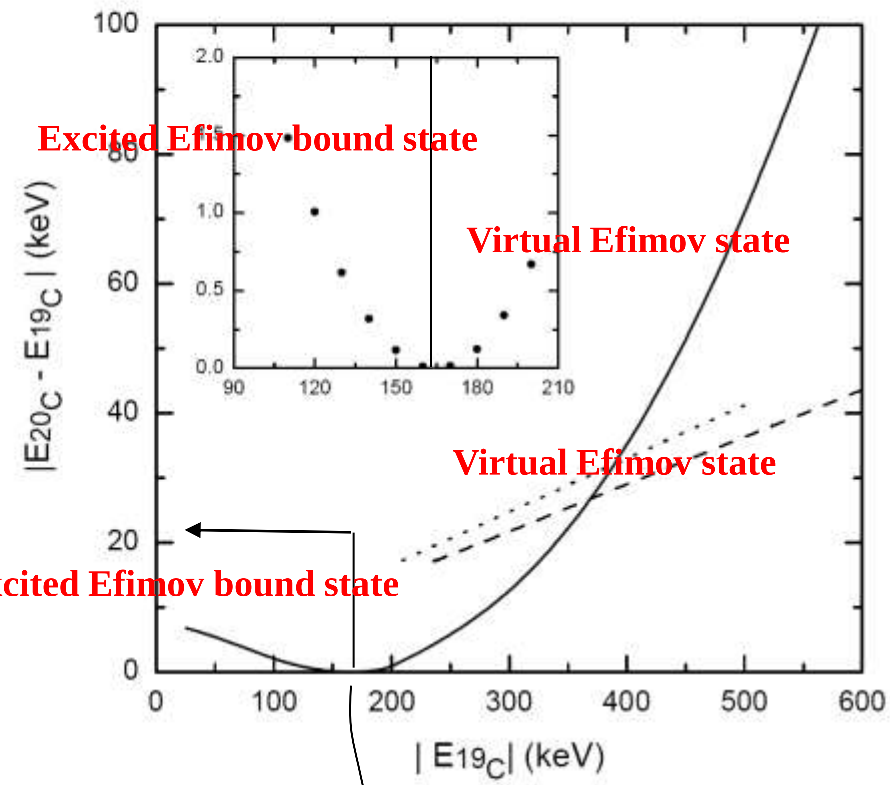
^{20}C can have a continuum resonance or virtual Efimov state?

Arora, Mazumdar, Bhasin PRC69 (2004)061301(R)   Mazumdar, Rau, Bhasin PRL97(2006)062503

Efimov state→resonance of n+^{19}C by changing Knc

Threshold for an excited Efimov state and trajectory: ^{20}C

Yamashita, Frederico,Tomio,
PRL99 (2007)269201 Comment on"Efimov States and their Fano resonances..."
& PLB660(2008)339



Efimov state in ^{20}C goes to a virtual state for $|E_{19C}| > 165keV$!

Critical value: $|E_{19C}|=165keV$

n-^{19}C scattering and Efimov physics

What to expect for s-wave scattering?
Look at doublet neutron-deuteron scattering…

Pole in s-wave kcot(δ) for n-d system ! Well known ~ 50 years

Delves' 60, Van oers & Seagrave' 67, Girard & Fuda' 78

$$k\cot\delta_0 = -A + Bk^2 - \frac{C}{1 + Dk^2},$$

The existence of the triton virtual state was found on the basis of the effective range expansion.

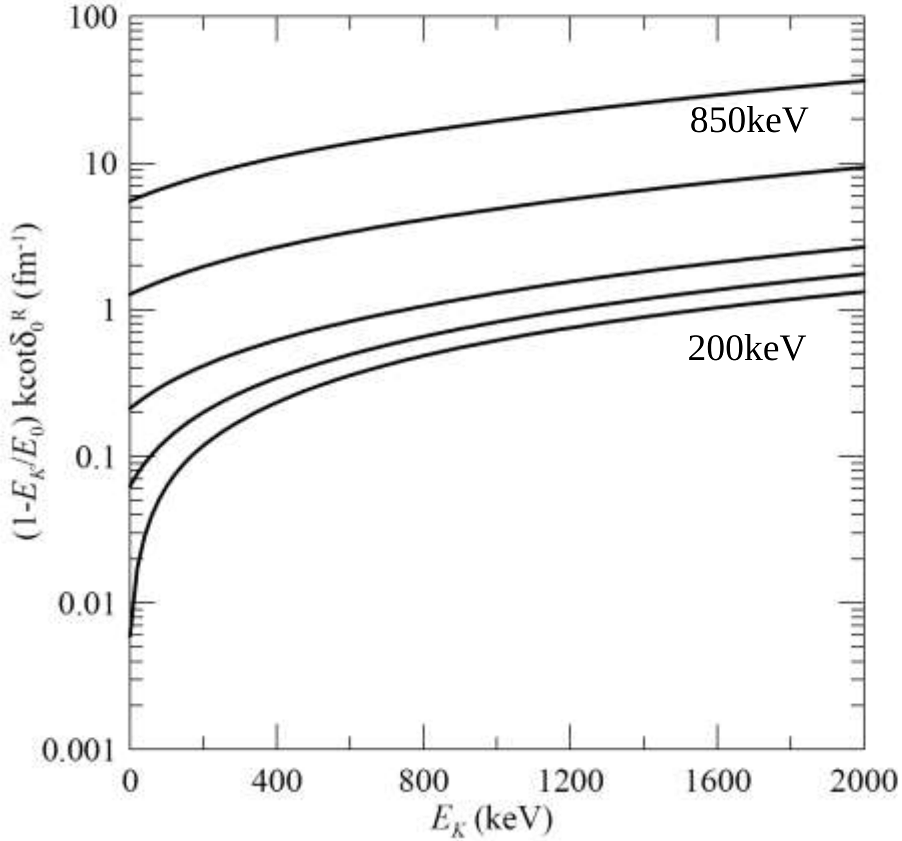
Universal property!

The atom-dimer (three-boson) scattering length is approximately given in Bratten and Hammer (Phys. Rep. 428 (2006) 259):

$$a_{AD} = (1.46 - 2.15\tan[s_0 ln(a\Lambda_*) + 0.09])a\ ,$$

where $s_0 = 1.00624$.

n-^{19}C scattering and Efimov physics



$$k\cot\delta_0^R = \frac{-a^{-1}_{n-{}^{19}C} + \beta\, E + \gamma\, E^2}{1 - E/E_0},$$

$\lvert E_{^{19}C}\rvert$(keV)	$(a_{n-^{19}C})^{-1}$ (fm^{-1})	β (fm.keV)$^{-1}$	γ (fm.keV^2)$^{-1}$	E_0 (keV)
200	-0.591 10^{-2}	5.685 10^{-4}	4.673 10^{-8}	1442.745
400	-0.624 10^{-1}	6.743 10^{-4}	8.821 10^{-8}	823.887
600	-2.118 10^{-1}	9.337 10^{-4}	1.464 10^{-7}	451.398
800	-1.268	3.11 10^{-3}	4.424 10^{-7}	114.976
850	-5.510	1.201 10^{-2}	1.641 10^{-6}	28.845

n-^{19}C scattering and Efimov physics

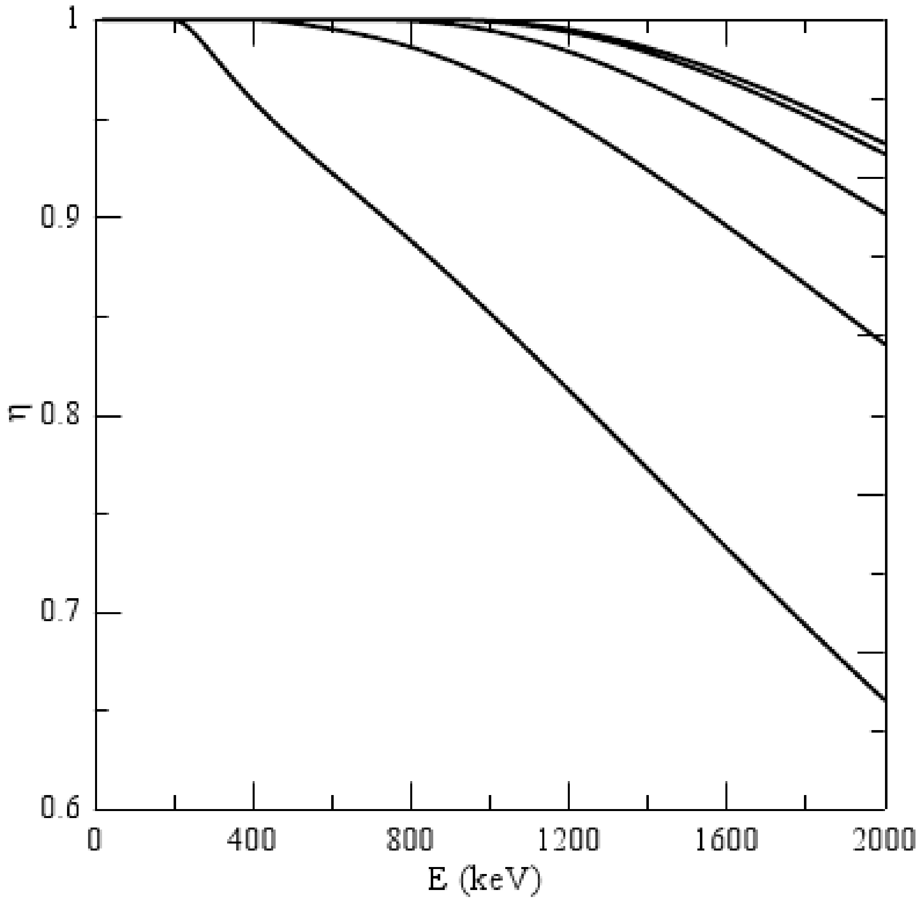
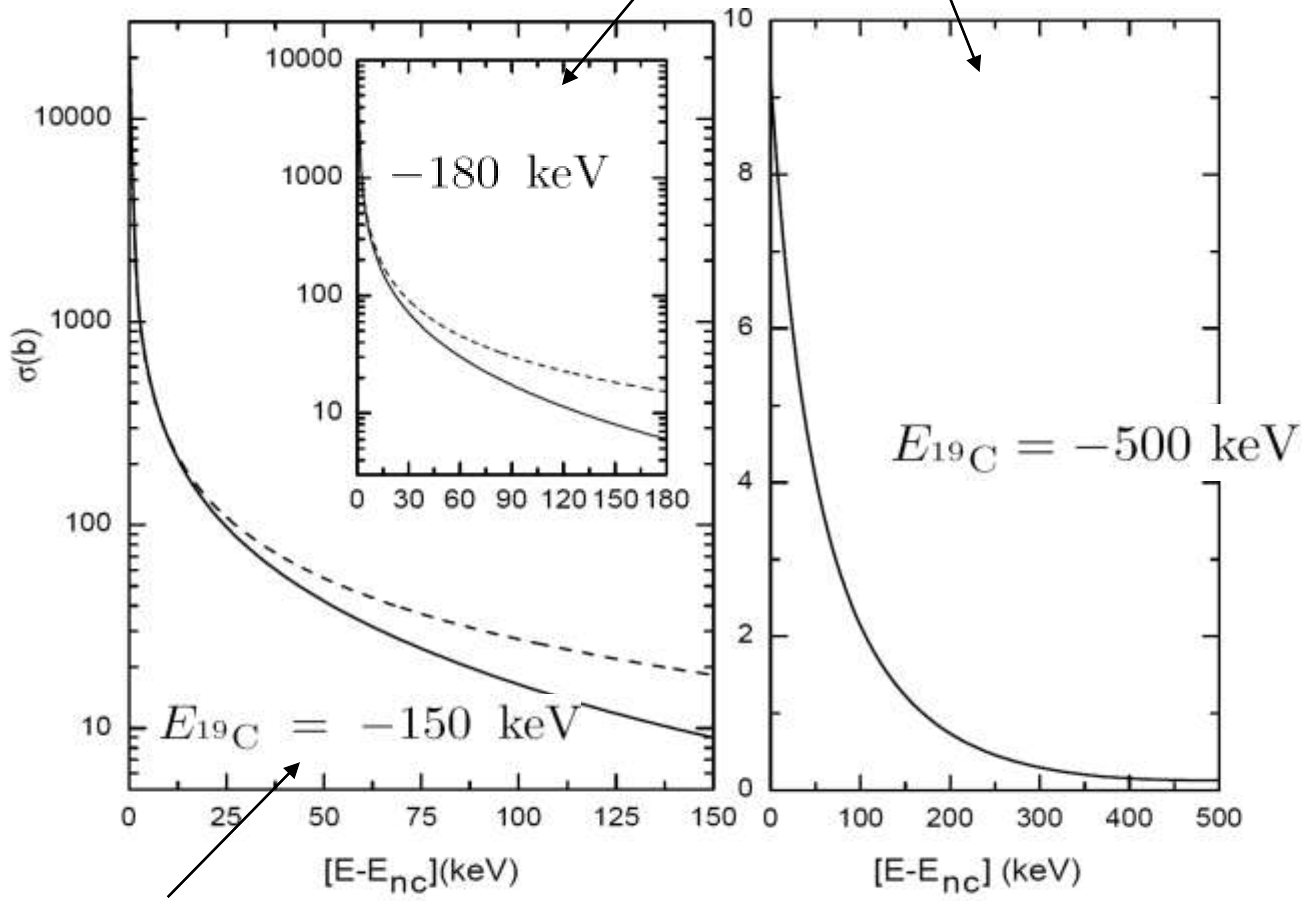


Fig. 4. s−wave absorption parameter as a function of the CM kinetic energy. From left to right the curves corresponds to the following ^{19}C energies: 200, 400, 600, 800 and 850 keV.

n-^{19}C scattering and Efimov physics

^{20}C has a virtual Efimov state

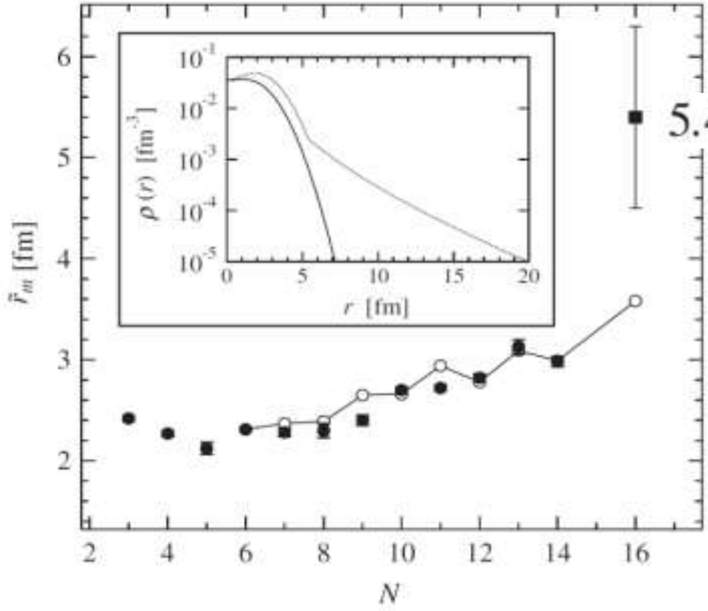


^{20}C has an excited bound Efimov state

^{22}C = n - n - ^{20}C

K. Tanaka *et al.*, Phys. Rev. Lett. **104** (2010) 062701

Reaction cross sections (σ_R) for ^{19}C, ^{20}C and the drip-line nucleus ^{22}C on a liquid hydrogen target have been measured at around 40A MeV by a transmission method. A large enhancement of σ_R for ^{22}C compared to those for neighboring C isotopes was observed. Using a finite-range Glauber calculation under an optical-limit approximation the rms matter radius of ^{22}C was deduced to be 5.4 ± 0.9 fm. It does not follow the systematic behavior of radii in carbon isotopes with $N \leq 14$, suggesting a neutron halo. It was found by an analysis based on a few-body Glauber calculation that the two-valence neutrons in ^{22}C preferentially occupy the $1s_{1/2}$ orbital.



5.4 ± 0.9 fm

$$S_{2n} = 420 \pm 940 \text{ keV}$$

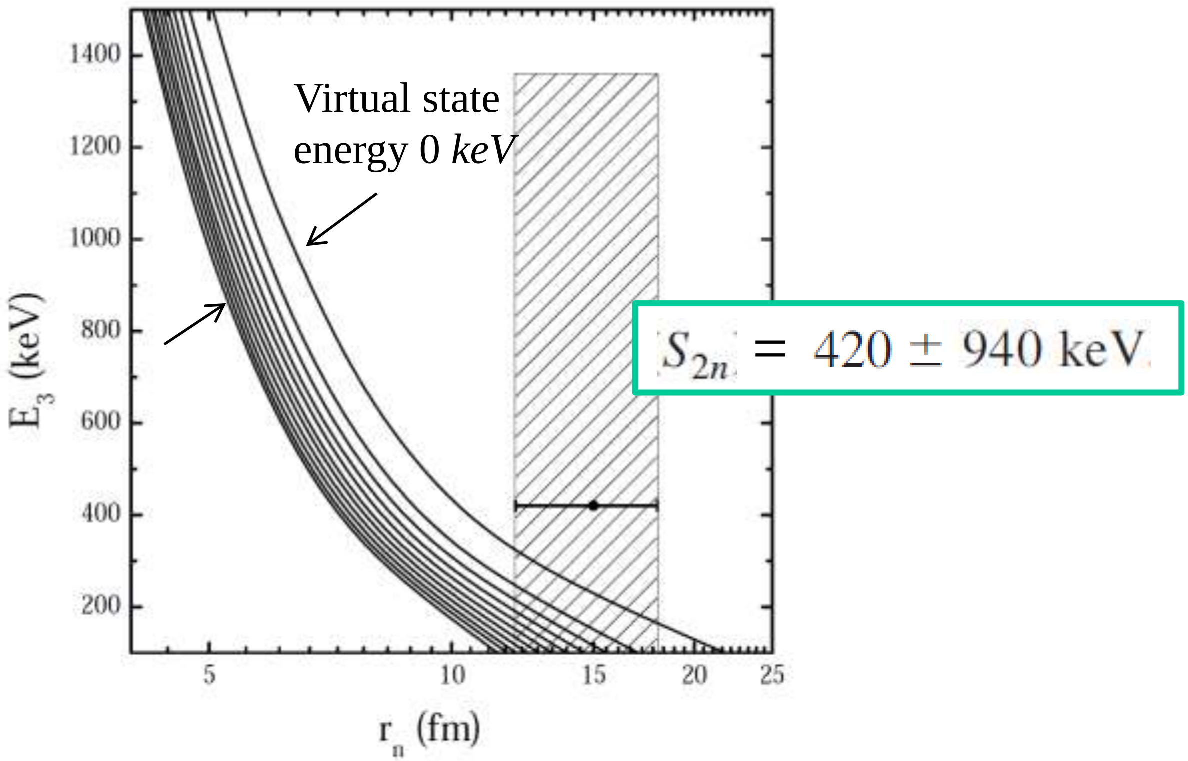
$$\tilde{r}_m^{22C} \equiv \langle (r_m^{22C})^2 \rangle^{1/2}$$

$$\tilde{r}_m^{20C} = 3 \; fm$$

$$\tilde{r}_n^{22C} = \sqrt{\frac{22}{2}} \sqrt{(\tilde{r}_m^{22C})^2 - \frac{20}{22}(\tilde{r}_m^{20C})^2} \approx 15 \pm 3 fm$$

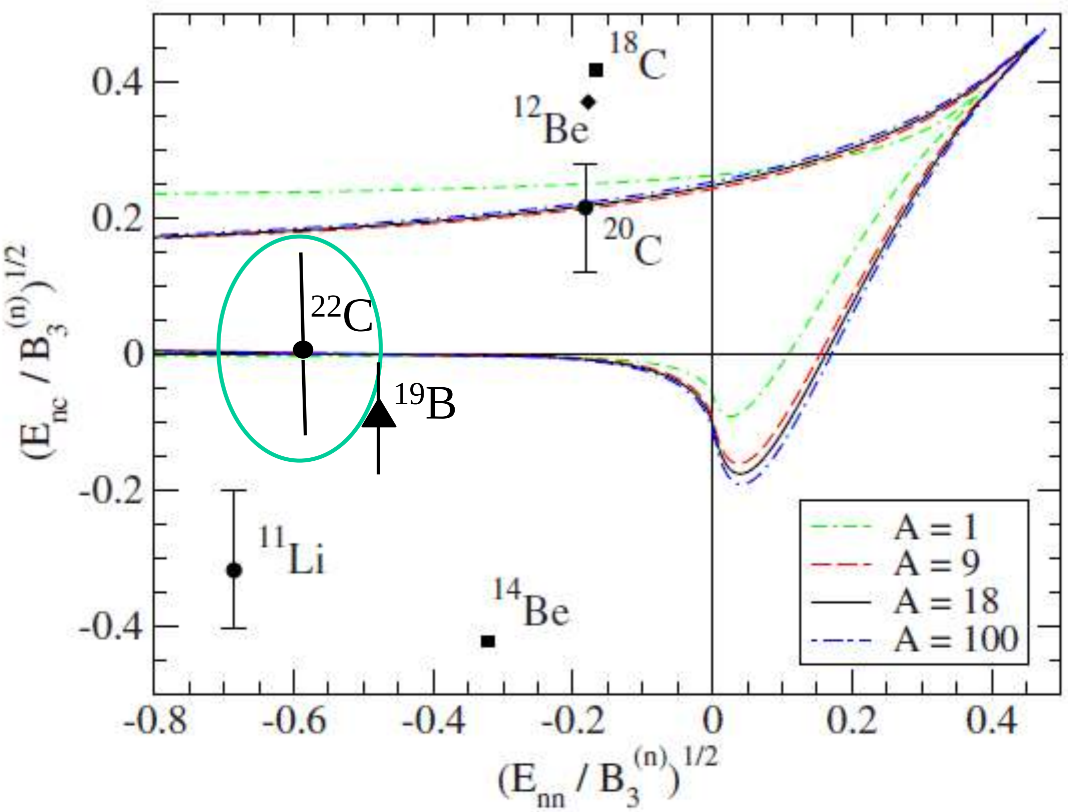
$^{22}C = n - n - {}^{20}C$

^{21}C virtual state energy 0, -10, -20 keV... Enn=-143KeV



Suggestion: is ^{21}C slightly bound?

$^{22}C = n - n - {}^{20}C$



From Canham and Hammer EPJ A 37 (2008) 367

^{21}C with a bound/virtual state with energy 0-10keV

→ It would be possible an excited Efimov state/continuum resonance!

Four-body scale: bosons

Thomas-Efimov effect: (low-mom. scale)/(high-mom. scale) →0

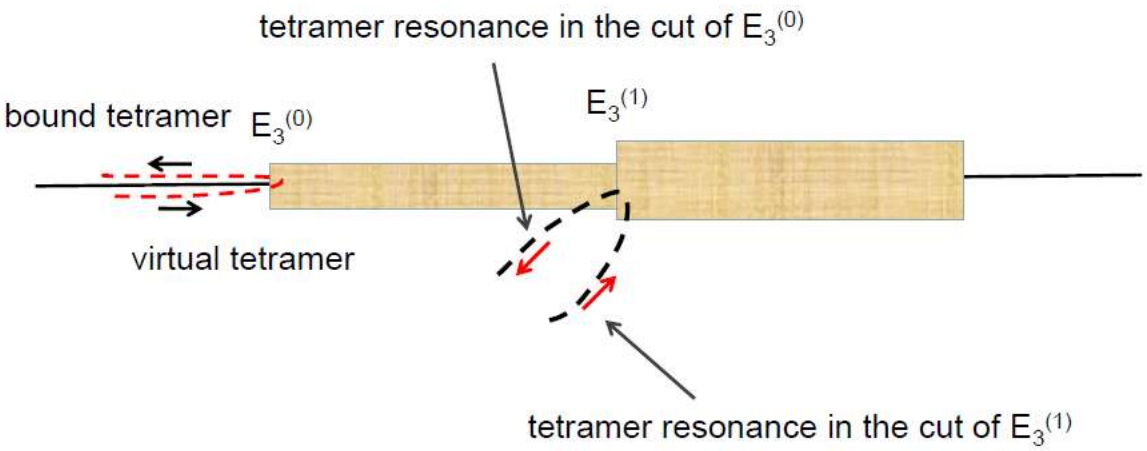
Yamashita, Tomio, Delfino & Frederico
Four-boson scale near a Feshbach resonance. Europhys. Lett.75 (2006) 555
•Tetramer ground state *moves* as a short-range scale collapses to zero with the trimer is fixed!
•coupling between a closed and open channels→ many-body forces in the open channel

•Tetramer is *fixed* by the trimer information:
Platter, Hammer, & Meissner,
Four-boson system with short-range interactions. Phys. Rev. A 70, 52101 (2004).
Stecher, D'Incao &Greene,
Signatures of universal four-body phenomena and their relation to the Efimov physics Nat.Phys. 5(09)417
Deltuva
Efimov physics in bosonic atom-trimer scattering, arXiv:1009.1295

•Empirical evidences for 3-body forces near a Feshbach resonance :
Pollack, Dries & Hulet (^{7}Li)
Universality in three- and four-body bound states of ultracold atoms. Science 326 (2009) 1683
Nakajima, Horikoshi, Mukaiyama, Naidon, & Ueda
Nonuniversal Efimov atom-dimer resonances in a three-component mixture of ^{6}*Li.*
Phys. Rev. Lett. 105, 023201 (2010)

4-body force near the Feshbach resonance?

Trajectory of four-boson bound states: one scenario…



?!

Summary and outlook

- Weakly bound & large systems: **few scales regime** in halo nuclei, molecules, trapped atoms
- Zero-range model n-n-c system:
  classification of weakly-bound systems
  threshold conditions for excited states and resonances
  borromean configuration: **Efimov state→ resonance**
  all-bound, Samba, Tango (at least one subsystem is bound): **Efimov state→ virtual state**
- ^{20}C **Efimov state→ virtual state** **E_{19C} > 165 keV**
- n+^{19}C scattering: **pole in the s-wave phase-shift**
- ^{22}C large nn halo S_{2n} ~ 400keV and ^{21}C virtual/bound state few keV
  → **Efimov continuum resonance/excited state**
- Exploration of universality in scattering, breakup of halo nuclei and large molecules
- Proton halo (Coulomb +contacts Higa, Hammer,Kolck NPA809(2008)171)
- P-wave (^{6}He=^{4}He+2n) Bertulani, Hammer,Kolck NPA 712 (2002) 37
- Neutron halo > 2: how many scales, how to describe few-neutrons+core dynamics...
- how many scales to describe few-particle weakly bound systems with short-range forces?

Backup: n-^{19}C scattering equations

$$\chi_n(\vec{q}) \equiv (2\pi)^3\delta(\vec{q}-\vec{k}_i) + 4\pi\frac{h_n(\vec{q};\mathcal{E}(k_i))}{q^2-k_i^2-\mathrm{i}\epsilon},$$

$$h_n^\ell(q;\mathcal{E}_i) = \mathcal{V}^\ell(q,k_i;\mathcal{E}_i) + \frac{2}{\pi}\int_0^\infty dk\,k^2\,\frac{\mathcal{V}^\ell(q,k;\mathcal{E}_i)\,h_n^\ell(k;\mathcal{E}_i)}{k^2-k_i^2-\mathrm{i}\epsilon}.$$

$$\mathcal{V}^\ell(q,k;\mathcal{E}) \equiv \pi\frac{(A+1)}{A+2}\left[K_2^\ell(q,k;\mathcal{E}) + \int_0^\infty dk'k'^2 K_1^\ell(q,k';\mathcal{E})\tau_{nn}(k';\mathcal{E})K_1^\ell(k,k';\mathcal{E})\right]$$

$$\tau_{nn}(q;\mathcal{E}) \equiv \frac{-2}{\pi}\left[\sqrt{|\varepsilon_{nn}|}+\sqrt{\frac{A+2}{4A}q^2-\mathcal{E}}\right]^{-1}, \quad \bar{\tau}_{nc}(q;\mathcal{E}) \equiv \frac{-1}{\pi}\left(\frac{A+1}{2A}\right)^{\frac{3}{2}}\left(\sqrt{|\varepsilon_{nc}|}+\sqrt{\frac{(A+2)q^2}{2(A+1)}-\mathcal{E}}\right),$$

$$K_{i=1,2}^\ell(q,k;\mathcal{E}) \equiv G_i^\ell(q,k;\mathcal{E}) - \delta_{\ell 0}\,G_i^\ell(q,k;-\mu^2),$$

$$G_i^\ell(q,k;\mathcal{E}) = \int_{-1}^{1} dy\,\frac{P_\ell(y)}{\mathcal{E}-\frac{A+1}{A+A^{i-1}}q^2-\frac{A+1}{2A}k^2-\frac{kqy}{A^{i-1}}+\mathrm{i}\epsilon}.$$