Towards an effective theory for heavy nuclei
Model-independent description of nuclear rotation

Thomas Papenbrock

THE UNIVERSITY OF TENNESSEE 1794

and OAK RIDGE NATIONAL LABORATORY

presently at

GSI theory

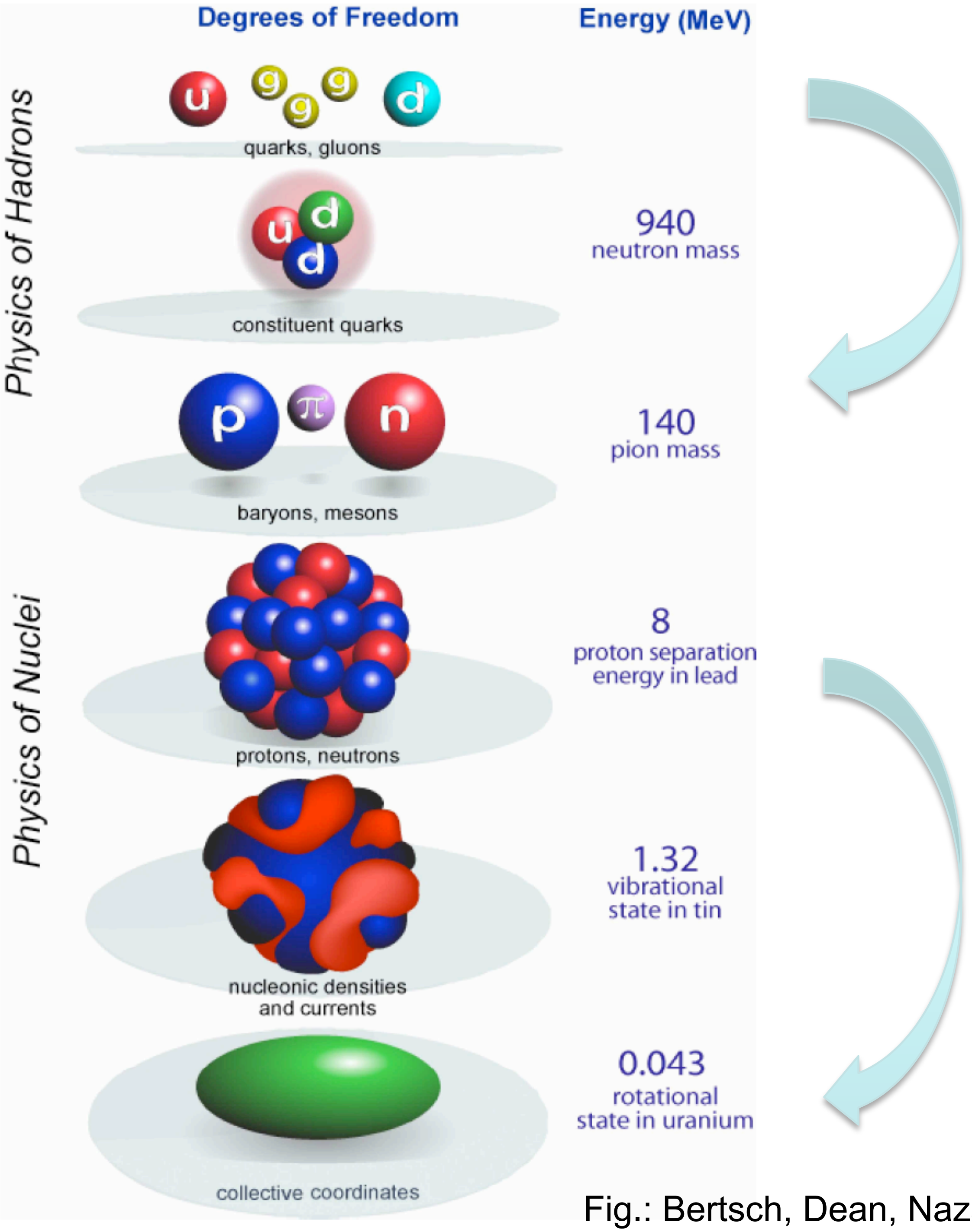
TECHNISCHE UNIVERSITÄT DARMSTADT

The limits of existence of light nuclei

ECT* Trento, October 25-30 2010

Research partly funded by the US Department of Energy and Alexander von Humboldt Stiftung

Energy scales and relevant degrees of freedom



Chiral effective field theory

Effective Theory

Fig.: Bertsch, Dean, Nazarewicz, SciDAC review (2007)

Models for nuclear rotations

1. Bohr Hamiltonian (1952), Bohr & Mottelson’s collective model (1953), Nobel Prize 1975
2. General collective model “Frankfurt model” [Gneuss, Mosel & Greiner (1969); Hess, Maruhn, Greiner (1981)]
3. Arima & Iachello’s Interacting Boson Model (1976)

- The existing models describe vast sets of data very well.
- They are difficult to generalize in a tractable way due to the difficulties in coupling of angular momenta and the computation of matrix elements.
- Renewed interest in computationally tractable models [M.A. Caprio, Phys. Rev. C 68, 054303 (2003); D.J. Rowe, Nucl. Phys. A 735, 372 (2004)]

Approach within an effective field theory possible.

Construction of an EFT

1. Identify the relevant degrees of freedom for the resolution scale of interest:
   Quadrupole phonons
2. Identify the relevant symmetries of low-energy nuclear physics and investigate if and how they are broken:
   Spontaneously broken rotational symmetry
3. Construct the most general Lagrangian consistent with those symmetries and the symmetry breaking.
   Nonlinear realization of rotational symmetry
4. Design an organizational scheme (power counting) that can distinguish between more and less important contributions:
   Separation of scale between rotational and vibrational modes

Useful references:

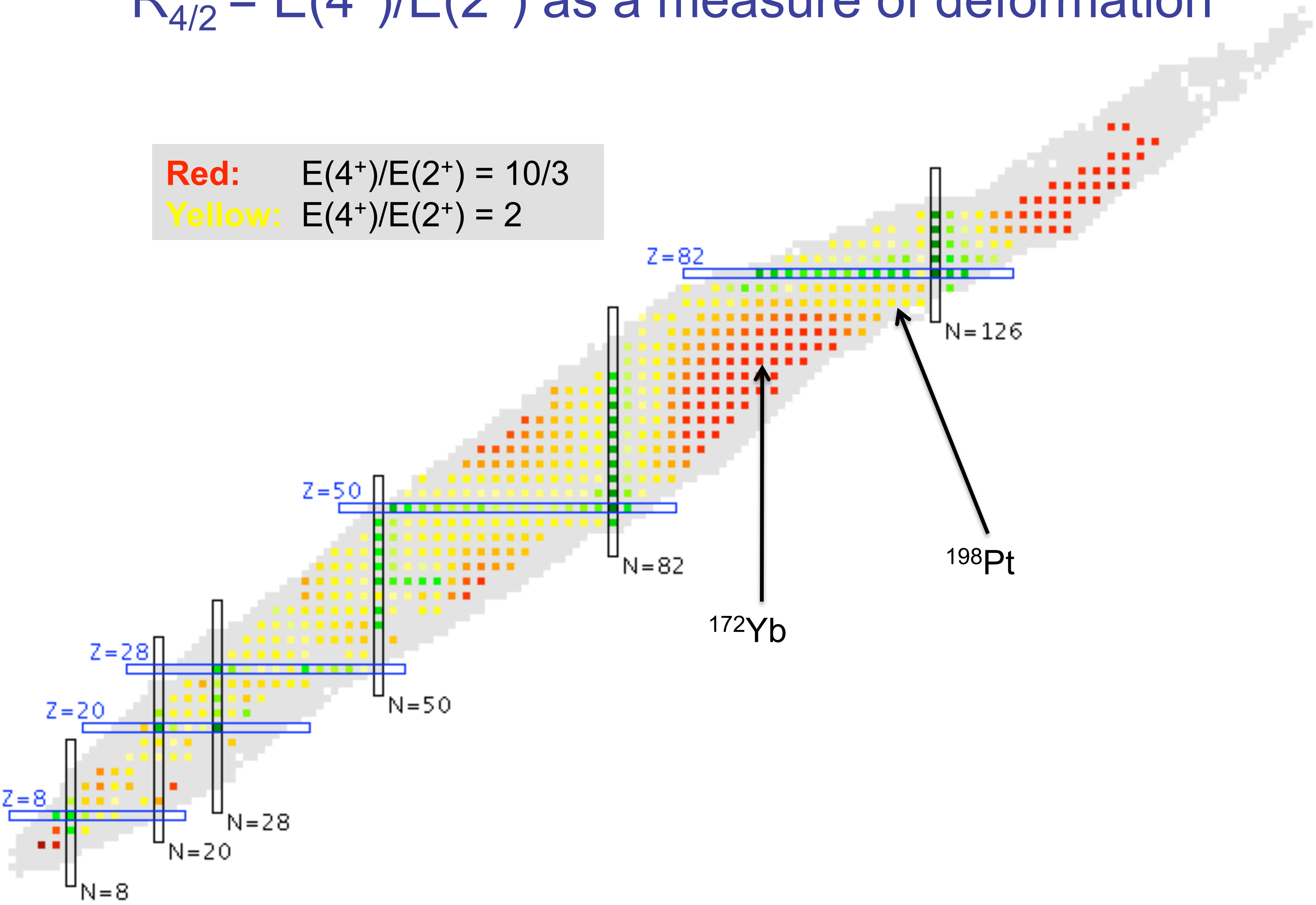
S. Weinberg, *The Quantum Theory of Fields,* Vol.II, chap. 19

H. Leutwyler, arXiv:hep-ph/9311264

C. P. Burgess, Physics Reports 330 (2000) 193

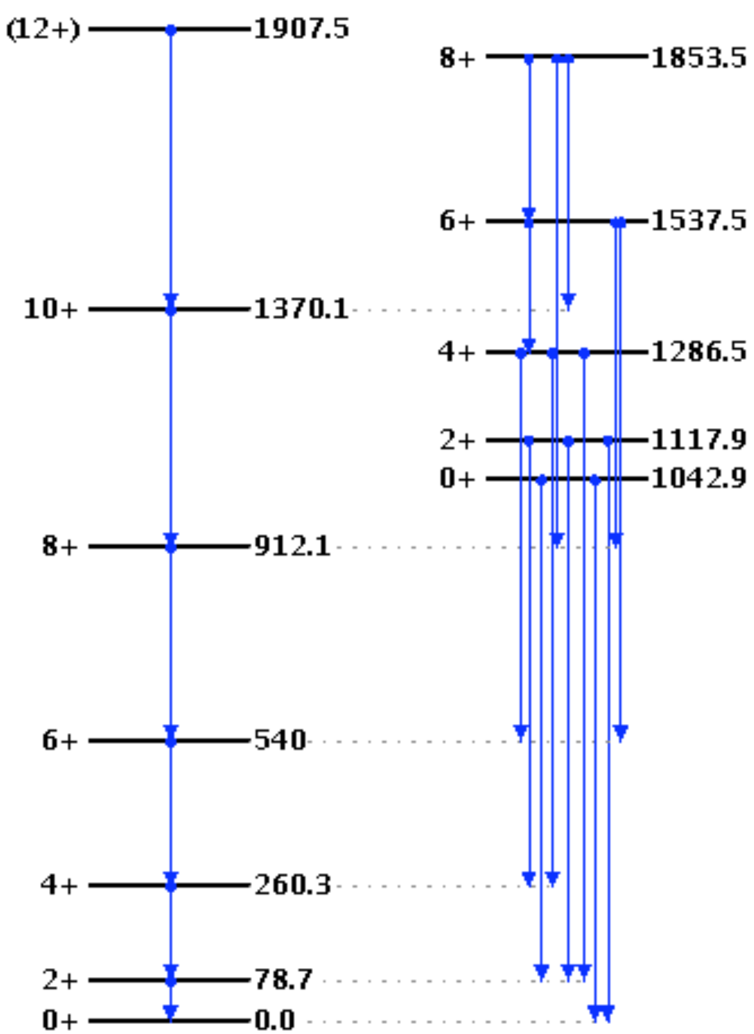
$R_{4/2} = E(4^+)/E(2^+)$ as a measure of deformation

Red: $E(4^+)/E(2^+) = 10/3$
Yellow: $E(4^+)/E(2^+) = 2$

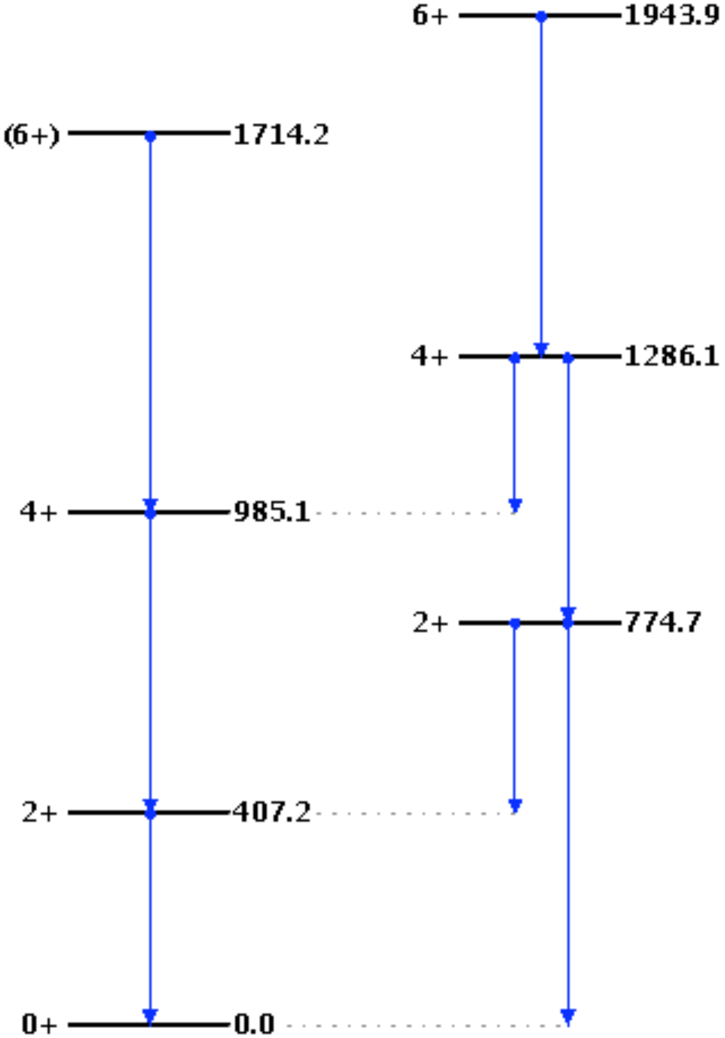


1. Identify relevant degrees of freedom

1. Identify relevant degrees of freedom

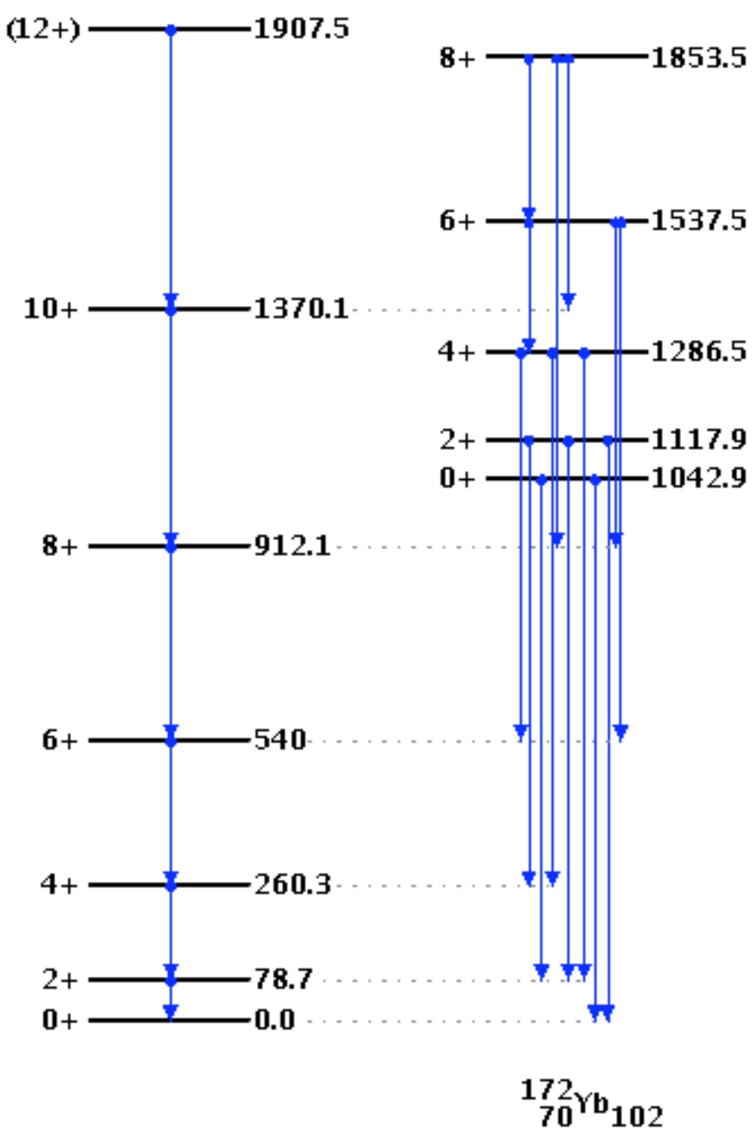


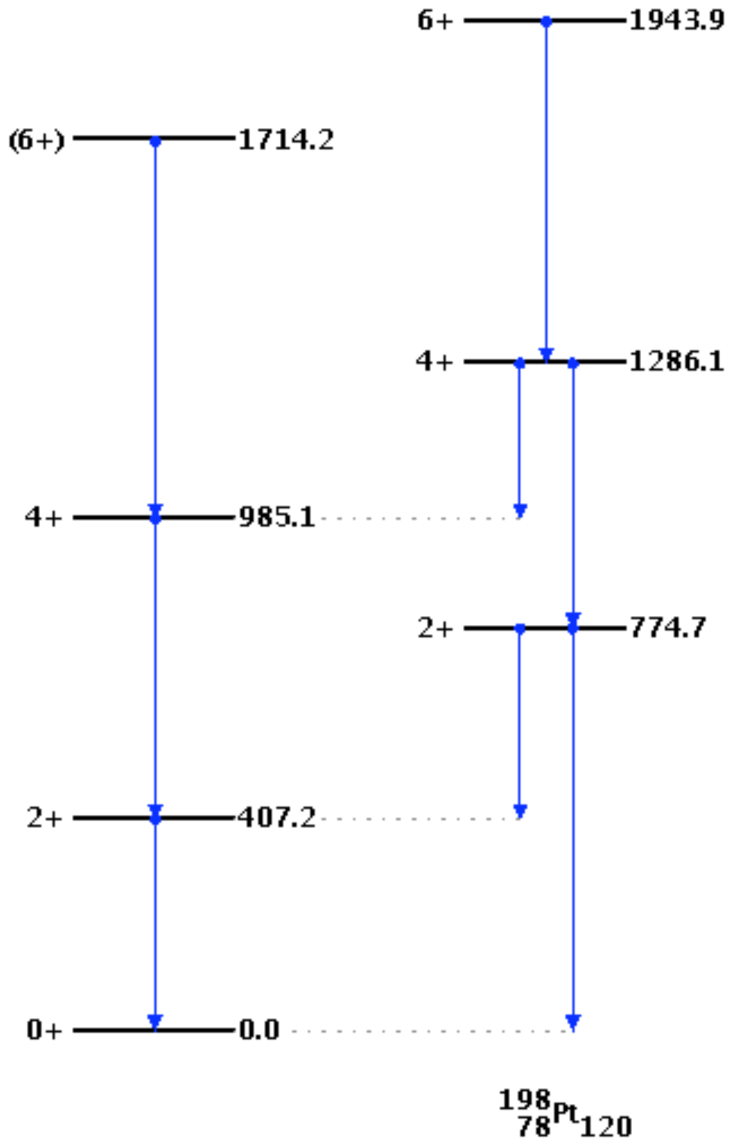
$^{172}_{70}\mathrm{Yb}_{102}$



$^{198}_{78}\mathrm{Pt}_{120}$

1. Identify relevant degrees of freedom

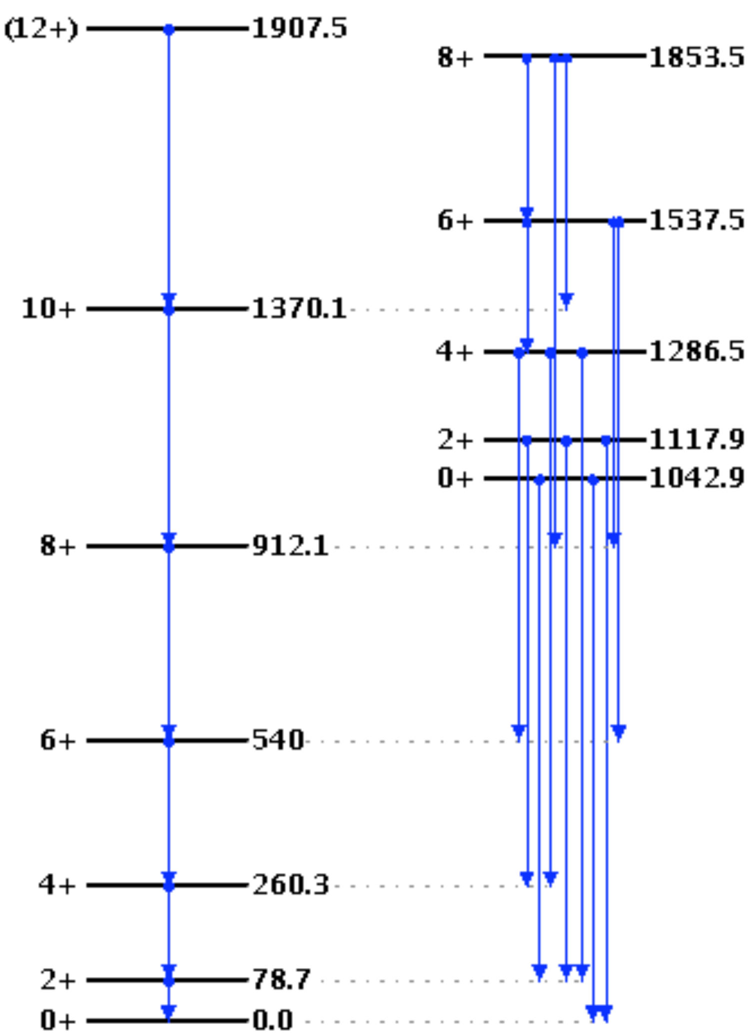




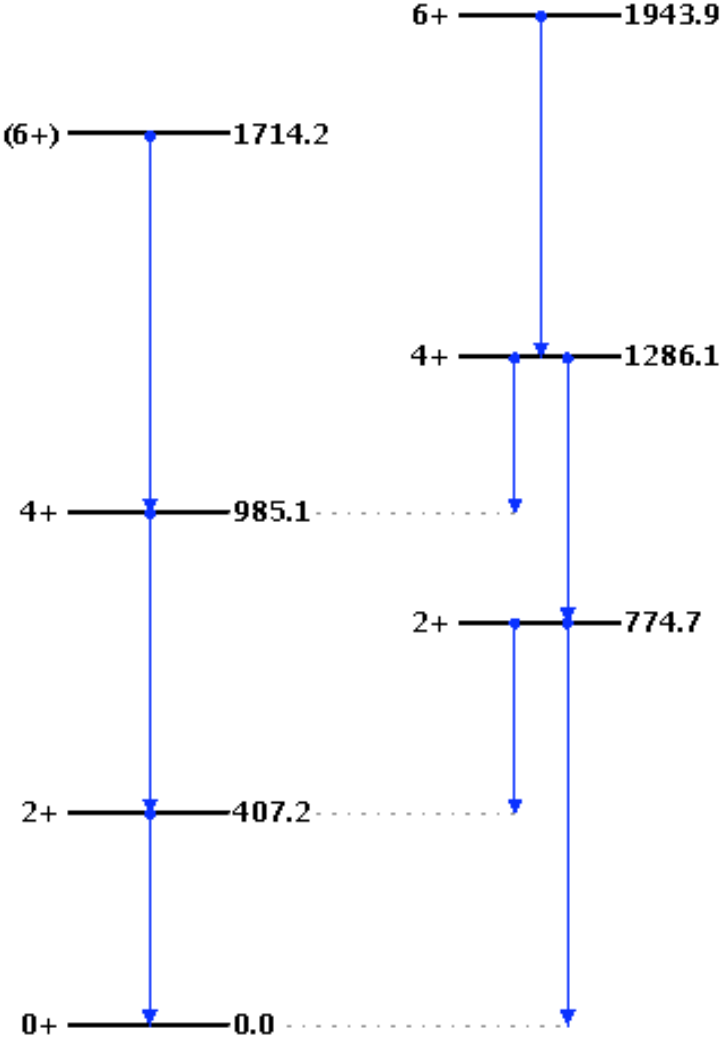
Quadrupole degrees of freedom describe spins and parity of low-energy spectra

2. Identify relevant symmetries and symmetry breaking

2. Identify relevant symmetries and symmetry breaking

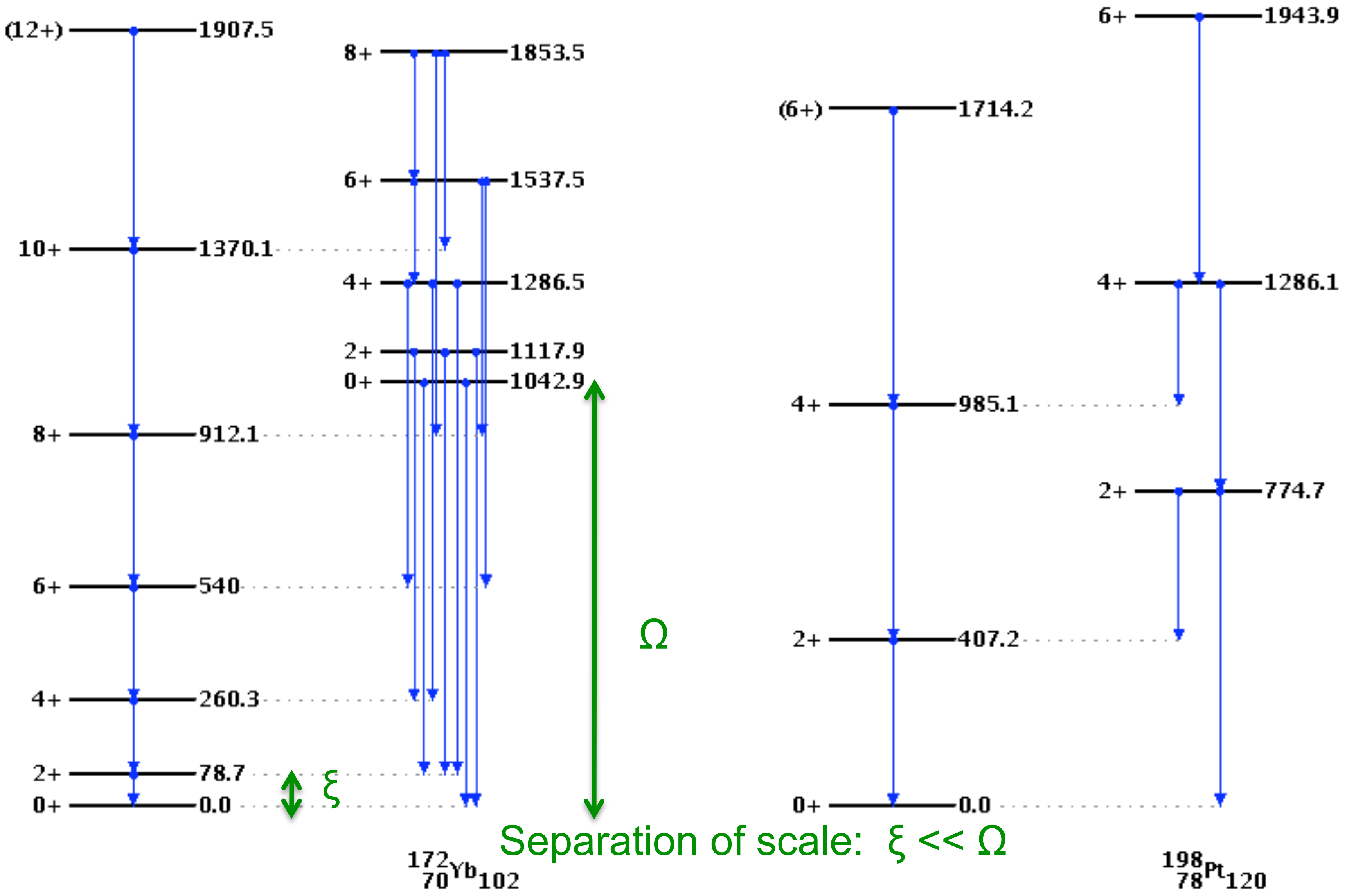


$^{172}_{70}\mathrm{Yb}_{102}$



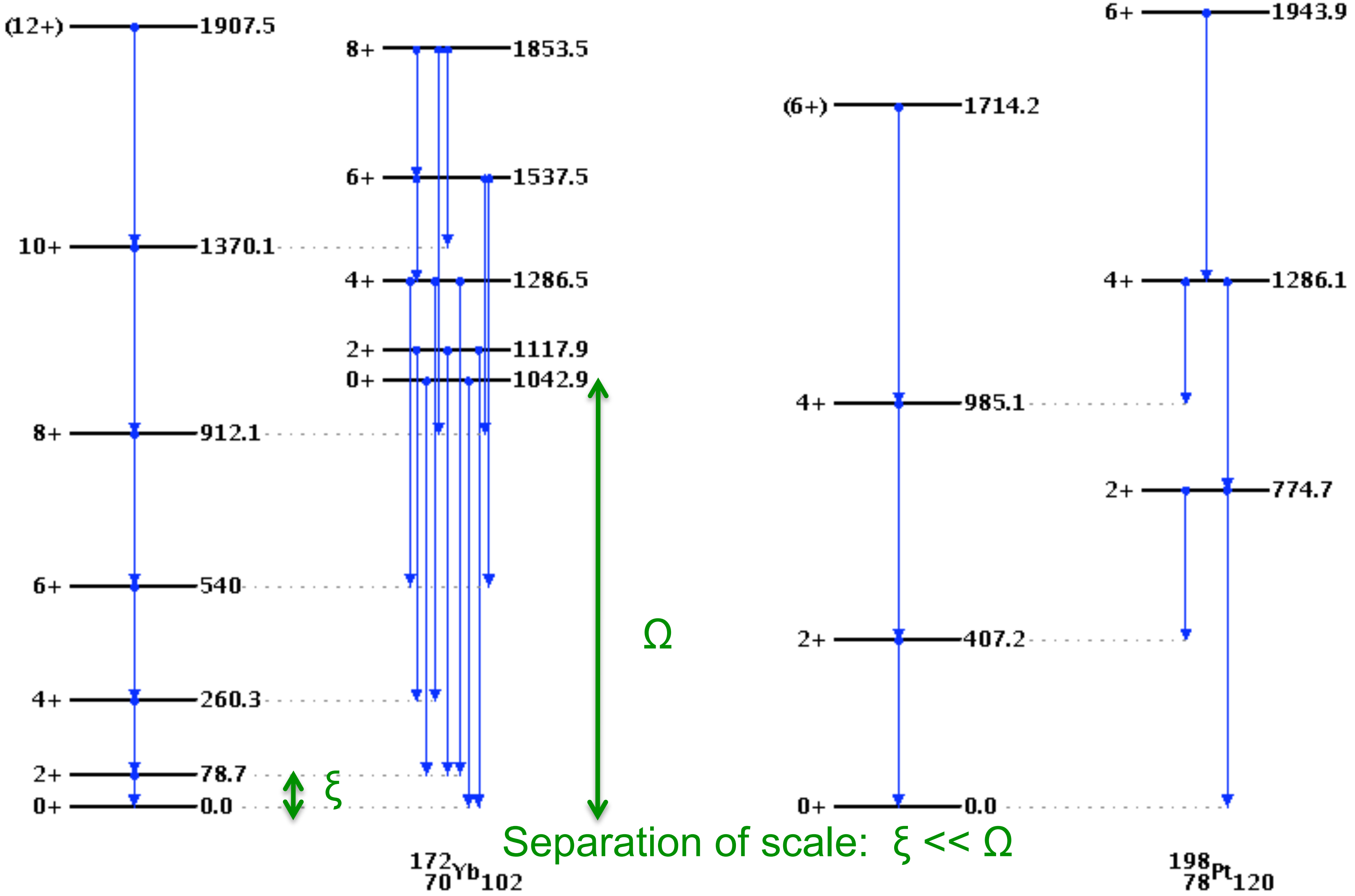
$^{198}_{78}\mathrm{Pt}_{120}$

2. Identify relevant symmetries and symmetry breaking



Separation of scale: ξ << Ω

2. Identify relevant symmetries and symmetry breaking

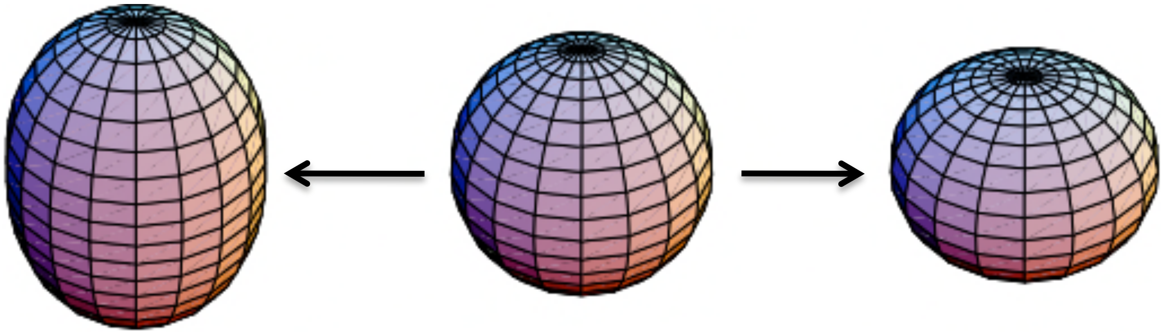


Separation of scale: ξ << Ω

Symmetry: Rotational invariance

Very low energy excitations (Nambu-Goldstone modes) indicate spontaneous symmetry breaking

Spontaneous breaking of rotational symmetry



Rotational symmetry → Axial symmetry
SO(3) → SO(2)

3 generators 1 generator

There will be 3-1=2 Nambu-Goldstone bosons

“*In a general theory of rotation, symmetry plays a central role. Indeed, the very occurrence of collective rotational degrees of freedom may be said to originate in a breaking of rotational invariance, which introduces a “deformation” that makes it possible to specify an orientation of the system. Rotation represents the collective mode associated with such a spontaneous symmetry breaking (Goldstone boson).*” Aage Bohr, Nobel Lecture (1975)

3. Construct the most general Hamiltonian consistent with the symmetry and the symmetry breaking:

Nonlinear realization of (rotational) symmetry

- Ground state exhibits only axial symmetry (symmetry group is SO(2))
- NG modes (Euler angles α and β) parameterize the coset SO(3)/SO(2))
- Building blocks with proper transformation properties:

1. E_x and E_y transform as the x and y-components of a vector under rotations.

$$\begin{aligned} E_x &= \dot{\alpha}\sin\beta \\ E_y &= -\dot{\beta} \end{aligned}$$

2. The "covariant derivative" D_t transforms as the z-component of a vector under rotations.

$$D_t \equiv \partial_t - iE_z J_z$$

$$E_z = -\dot{\alpha}\cos\beta$$

Any Lagrangian consisting of combinations of E_x, E_y, and D_t (acting on other fields) **that is formally invariant under SO(2)** (i.e. rotations around the z-axis) **is indeed invariant under SO(3)**.

Weinberg (1967); Coleman, Wess, Zumino (1969); Callan, Coleman, Wess & Zumino (1969).
Pedagogical reviews: S. Weinberg, The Quantum Theory of Fields, Vol.II, chap. 19: C. P. Burgess, Physics Reports 330 (2000) 193; H. Leutwyler, arXiv:hep-ph/9311264;T. Brauner, arXiv:1001.5212.

Physics of Nambu-Goldstone modes

Lagrangian

$$L_{\rm LO}^{(ee)} = \frac{C_0}{2}\left(E_x^2 + E_y^2\right)$$
$$= \frac{C_0}{2}\left(\dot\beta^2 + \dot\alpha^2 \sin^2\beta\right)$$

Hamiltonian

$$H = \frac{p_\beta^2}{2C_0} + \frac{p_\alpha^2}{2C_0 \sin^2\beta}$$

Quantization

$$p_\beta^2 = -\frac{1}{\sin\beta}\partial_\theta \sin\beta\partial_\beta \ ,$$
$$p_\alpha = -i\partial_\alpha \ .$$

Spectrum

$$\hat{H} Y_{lm}(\beta,\alpha) = \frac{l(l+1)}{2C_0} Y_{lm}(\beta,\alpha)$$

Rotational bands are quantized Nambu-Goldstone modes.
Low-energy constant C_0 is moment of inertia.

Nuclei with nonzero ground-state spins

A finite ground-state spin breaks time reversal invariance.
☺ Consider terms that are first order in the time derivative
☹ No such terms. BUT: under rotations, E_z changes by a total derivative.
Action remains essentially invariant (Wess-Zumino term)

Lagrangian
$$\begin{aligned} L_{\rm LO} &= L_{\rm LO}^{(ee)} + L_{\rm WZ} \\ &= \frac{C_0}{2}\left(\dot\beta^2 + \dot\alpha^2\sin^2\beta\right) - q\dot\alpha\cos\beta \end{aligned}$$

Hamiltonian
$$H_{\rm LO} = \frac{p_\beta^2}{2C_0} + \frac{(p_\alpha + q\cos\beta)^2}{2C_0\sin^2\beta}$$

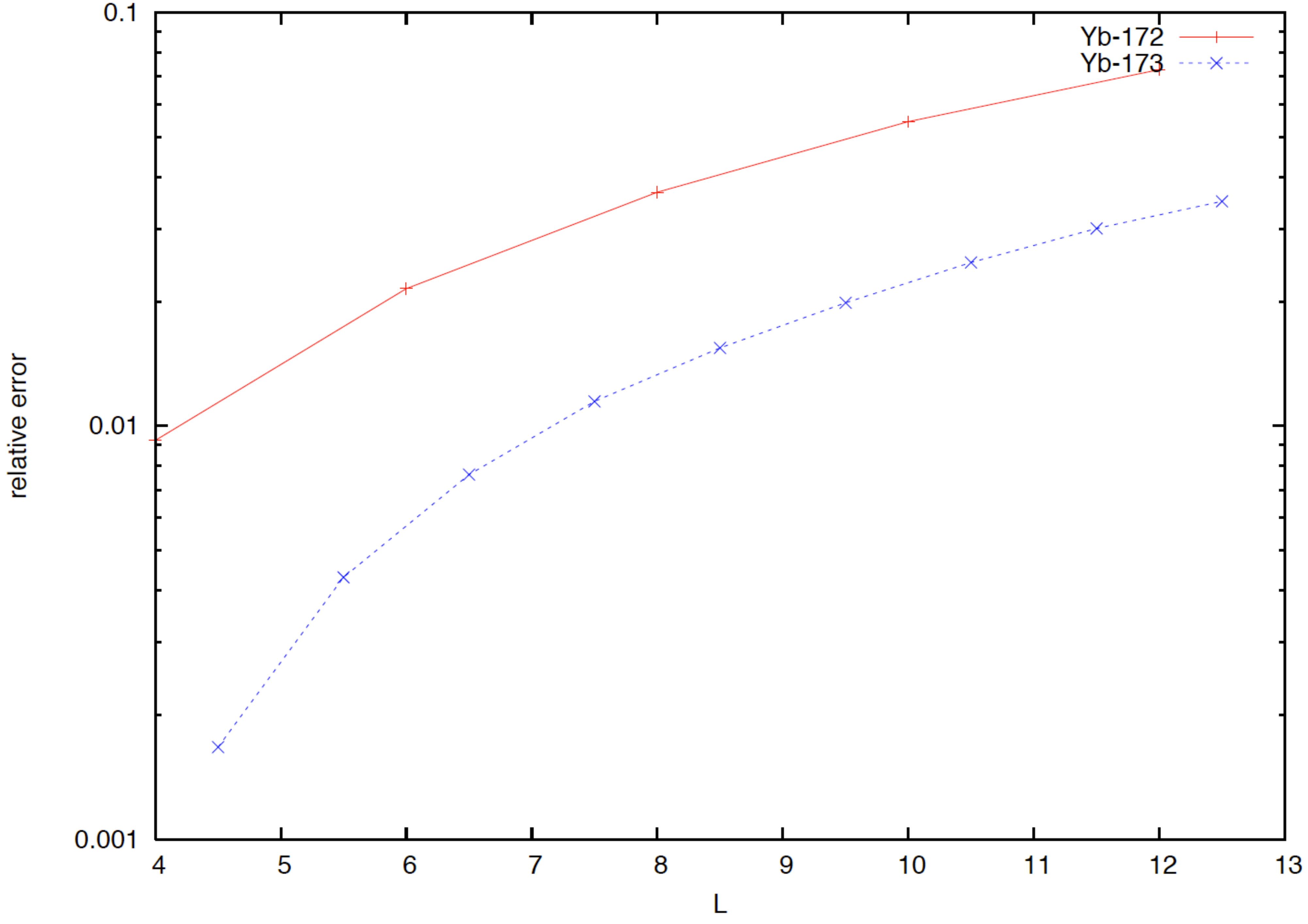
Eigenvalues and eigenfunctions (Identify q with ground-state spin!)

$$\hat H_{\rm LO}\, d^l_{mq}(\beta)e^{-i\alpha m} = E_{\rm LO}(q,l)d^l_{mq}(\beta)e^{-i\alpha m}$$

$$E_{\rm LO}(q,l) = \frac{l(l+1)-q^2}{2C_0} \qquad l = |q|, |q|+1, |q|+2, \ldots$$

$$D^l_{mq}(\alpha,\beta,\gamma) \equiv e^{-im\alpha}d^l_{mq}(\beta)e^{-iq\gamma}$$ (Wigner D functions)

Relative error in leading order



4. Power counting and beyond leading order

Estimates
(naïve dimensional analysis)
$H_{LO} \sim \xi$

$$\begin{aligned} C_0 &\sim \xi^{-1} \,, \\ p_\beta \sim p_\alpha \sim q &\sim \xi^0 \,, \\ \dot\beta \sim \dot\alpha \sim E_{x,y,z} &\sim \xi \,. \end{aligned}$$

Lagrangian at NLO

$$L_{\rm NLO} = L_{\rm LO} + \frac{C_2}{4}\left(E_x^2 + E_y^2\right)^2$$

Power counting

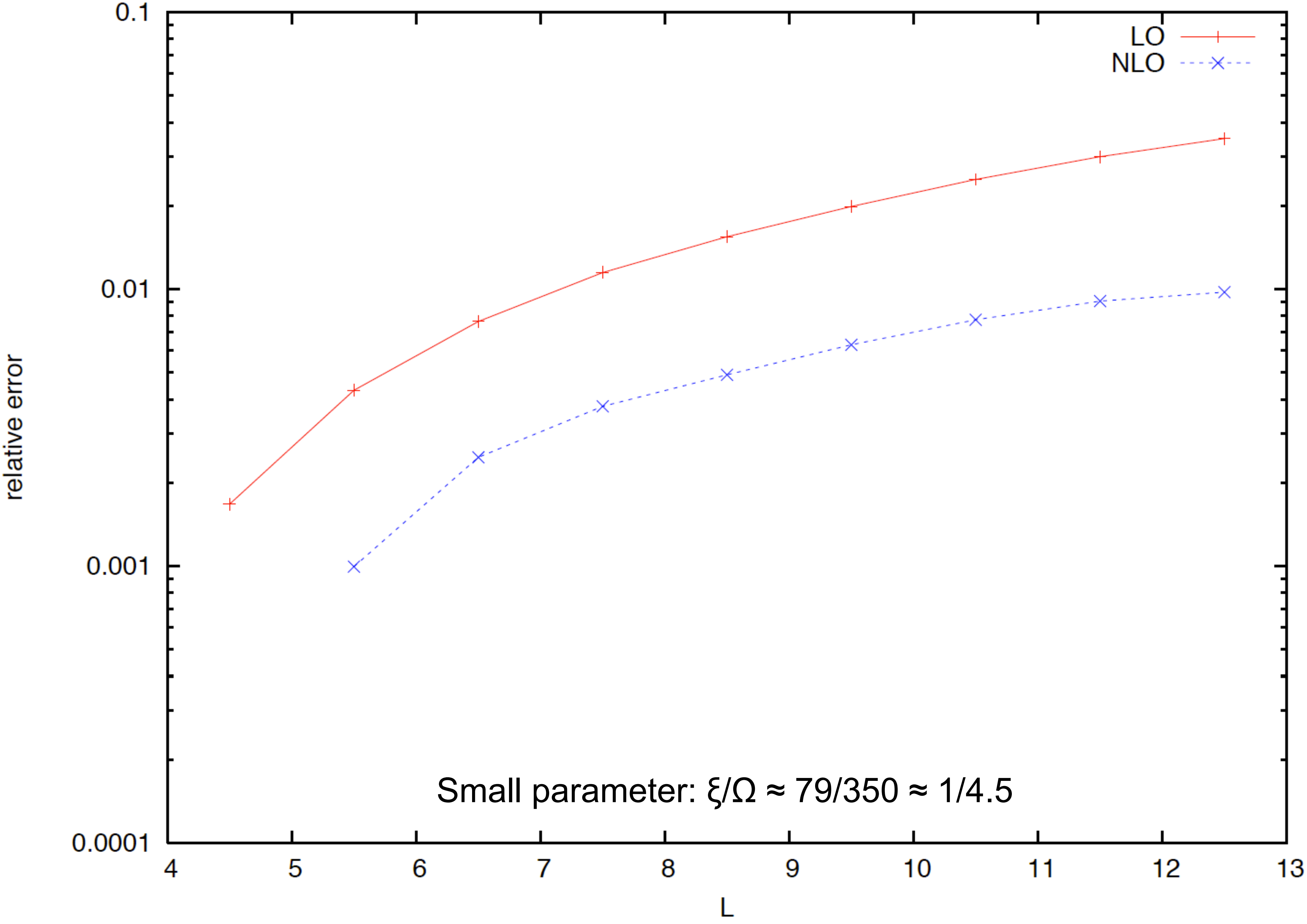
Main idea: higher-oder term due to neglected couplings between NG modes and vibrations. $[C_2/C_0]$= energy^{-2}

$$\frac{C_2}{C_0} \sim \Omega^{-2} \quad \Longrightarrow \quad \frac{C_2}{C_0}\left(E_x^2 + E_y^2\right) \sim \left(\frac{\xi}{\Omega}\right)^2 \ll 1$$

Spectrum: A $I(I+1)$ + $B(I(I+1))^2$ for even-even nuclei. (Bohr & Mottelson)
In general:

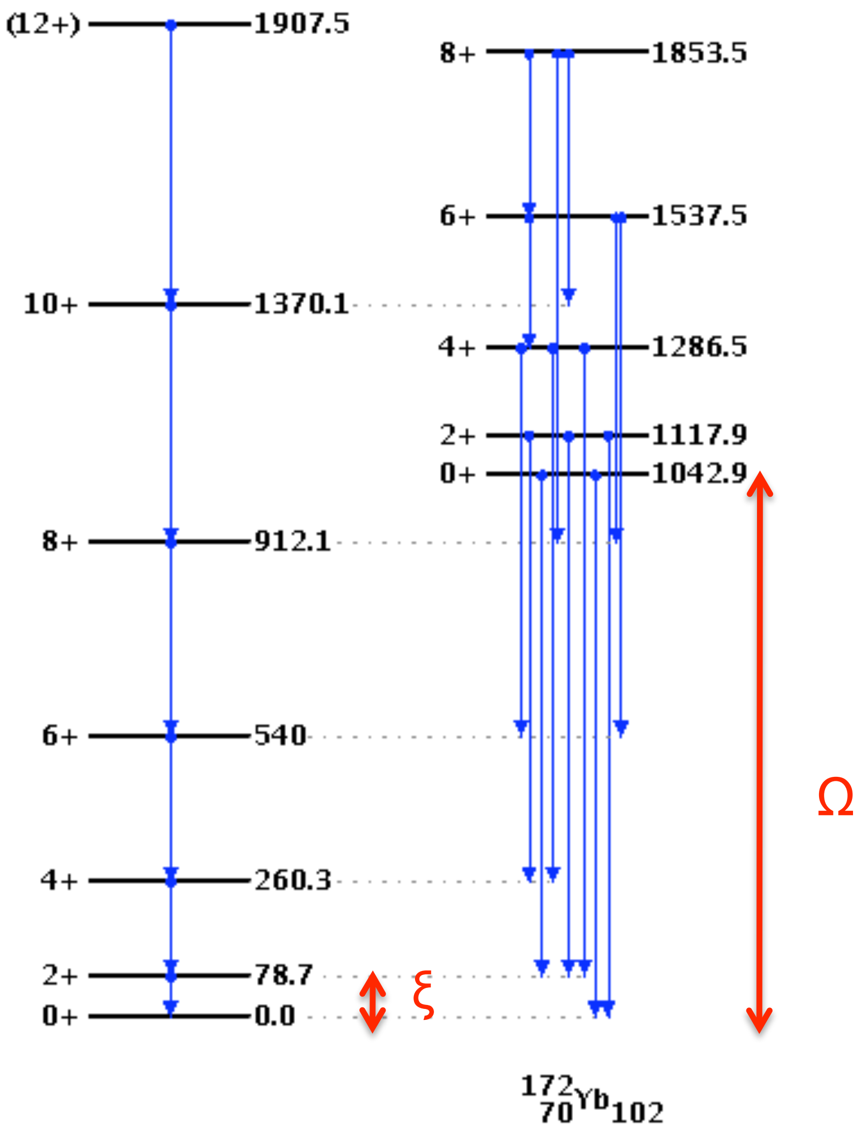
$$E_{\rm NLO} = \left(1 - \frac{C_2}{C_0^2} E_{\rm LO}\right) E_{\rm LO}$$

^{173}Yb: Relative error in LO and NLO



Small parameter: $\xi/\Omega \approx 79/350 \approx 1/4.5$

Beyond NG modes: coupling to vibrations



Higher energetic degrees of freedom need to be included.

Quadrupole field exhibits spontaneous symmetry breaking.

→5 DoF – 2NG = 3DoF

$$\phi = \begin{pmatrix} \phi_2 \\ 0 \\ \phi_0 \\ 0 \\ \phi_2^* \end{pmatrix}$$

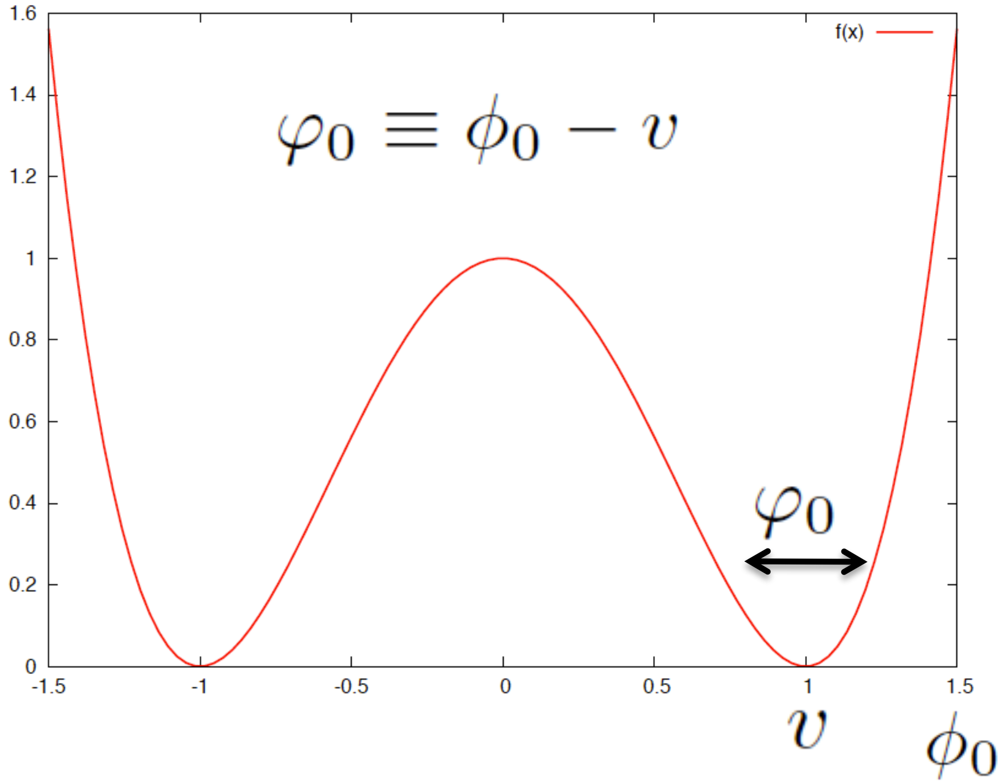
Separation of scale: $\xi << \Omega$

Couplings to vibrations: power counting

Low energy scale ξ
High energy scale Ω>>ξ

Dimensional analysis

$$\begin{aligned} v \sim \phi_0 &\sim \xi^{-1/2} \,, \\ \varphi_0 \sim \phi_2 &\sim \Omega^{-1/2} \,, \\ \dot{\varphi}_0 = \dot{\phi}_0 \sim \dot{\phi}_2 &\sim \Omega^{1/2} \,. \end{aligned}$$



Potential expanded around miniumum

$$V_2(\phi) = \frac{\omega_0^2}{2}(\phi_0 - v)^2 + \frac{\omega_2^2}{4}|\phi_2|^2$$

$$V = V_2 + \sum_{k+2l>2} v_{kl}\varphi_0^k|\phi_2|^{2l}$$

Power counting: large amplitudes $\varphi_0 \approx v$ restore rotational symmetry → breakdown of EFT

$$v_{kl}\varphi_0^k|\phi_2|^{2l} \sim \Omega\left(\frac{\xi}{\Omega}\right)^{l-1+k/2}$$

Leading order ~O(Ω)

Lagrangian at leading order:

$$L_{\mathrm{LO}} = \frac{1}{2}\dot{\varphi_0}^2 + |\dot{\phi}_2|^2 - \frac{\omega_0^2}{2}\varphi_0^2 - \frac{\omega_2^2}{4}|\phi_2|^2$$

$$H_{\mathrm{LO}} = \frac{1}{2}p_0^2 + \frac{1}{4}\left(p_{2r}^2 + p_{2i}^2\right) + \frac{\omega_0^2}{2}\varphi_0^2 + \frac{\omega_2^2}{4}\left(\phi_{2r}^2 + \phi_{2i}^2\right)$$

Spectrum

$$E(n_0, n_2, m_l) = \omega_0\left(n_0 + \frac{1}{2}\right) + \frac{\omega_2}{2}(2n_2 + |m_l| + 1)$$

Leading order yields the band heads

Lagrangian of E_x, E_y, $D_t\varphi_0$, $D_t\phi_2$, φ_0, ϕ_2, needs to be formally invariant under SO(2) (axial symmetry only).

Next-to-leading order ~O(ξ)

Lagrangian
$$L_{\rm NLO} = L_{\rm LO} + \frac{3}{2}v^2\left(E_x^2 + E_y^2\right) - 4E_z {\rm Im}\left(\dot\phi_2\phi_2^*\right) - \sum_{k+2l=3,4} v_{kl}\varphi_0^k|\phi_2|^{2l} \; .$$

Hamiltonian (kinetic energy)

$$H_{\rm NLO} = H_{\rm LO} + \frac{1}{6v^2}\left(p_\beta^2 + \frac{1}{\sin^2\beta}\left[p_\alpha^2 + 2p_\alpha l_z\cos\beta\right]\right)$$

Spectrum: a rotational band on every vibrational band head

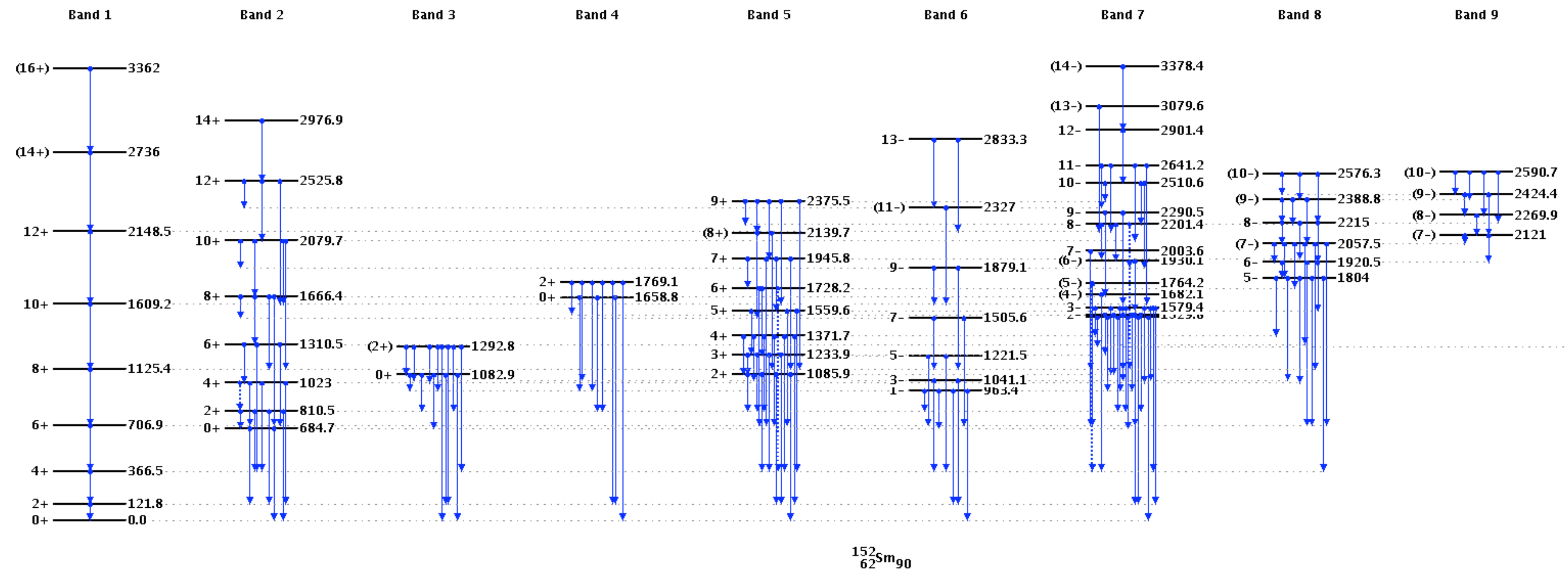
$$E(n_0,n_2,m_l,l) = \omega_0\left(n_0+\frac{1}{2}\right) + \frac{\omega_2}{2}(2n_2+|m_l|+1) + \frac{1}{6v^2}\left(l(l+1)-(2m_l)^2\right)$$

Corrections ~ξ of band heads due to anharmonicities in the potential neglected.

In next-to-leading order, the results of the rotational-vibrational model are reproduced.

Summary & Challenges

1. Effective theory for nuclear rotation
2. Based on separation of scale between rotations (NG modes) and vibrations
3. Implemented via nonlinear realization of rotational symmetry
4. To do:
    - Inclusion of fermionic degrees of freedom
    - Inclusion of octupole phonons
    - Coupling to EM fields; computation of transition strengths
    - …
    - challenge: “transitional” nuclei such as ^{152}Sm

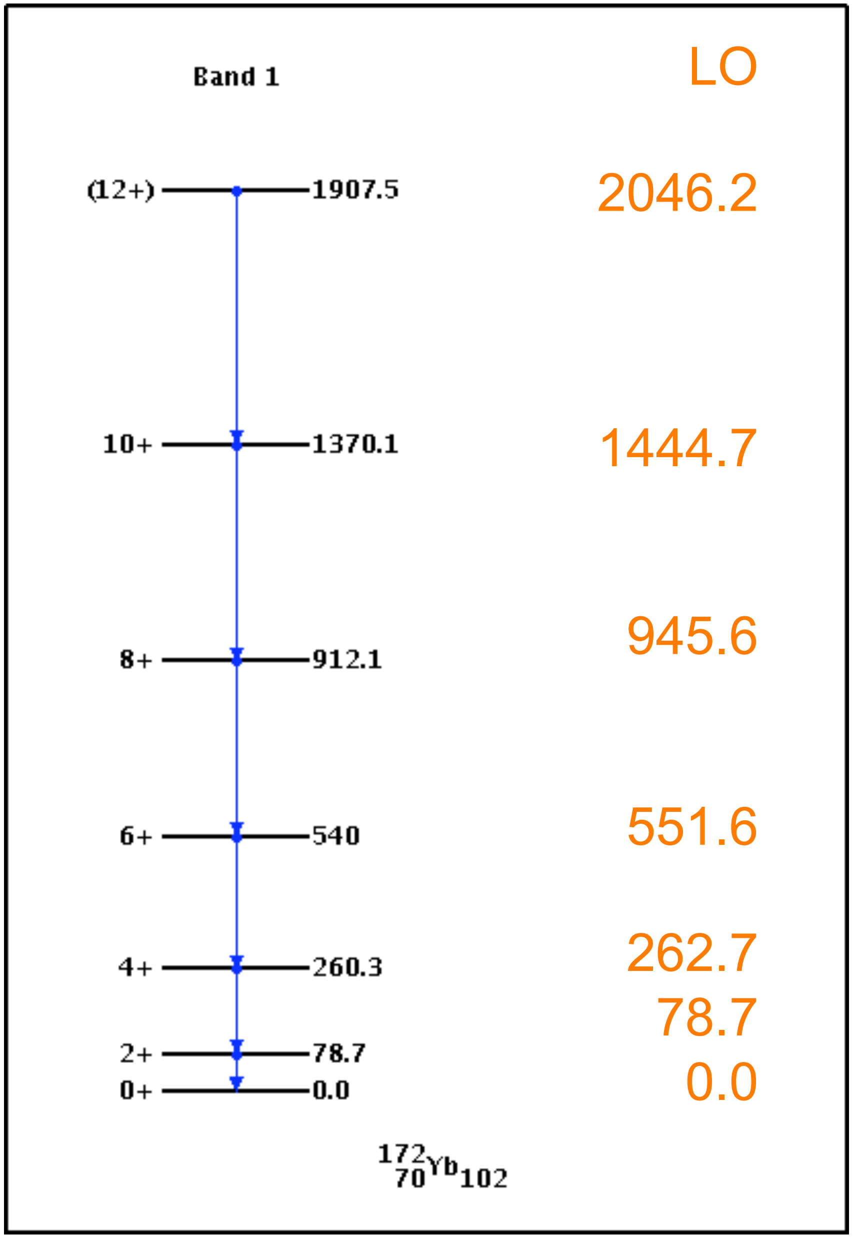


$^{152}_{62}\mathrm{Sm}_{90}$

Models vs. EFT

Model	**EFT**
Inituitive, tractable, beautiful	Systematic parametrization of ignorance
Coupling to good angular momentum	Nonlinear realization of rotational symmetry yields Lagrangians that are formally (only) axially symmetric
Three analytically known limits; no power counting	Power counting and analytical results at NLO
Rotors and vibrators within one model	Deformed nuclei

Comparison with data (energies in keV)



LO

2046.2

1444.7

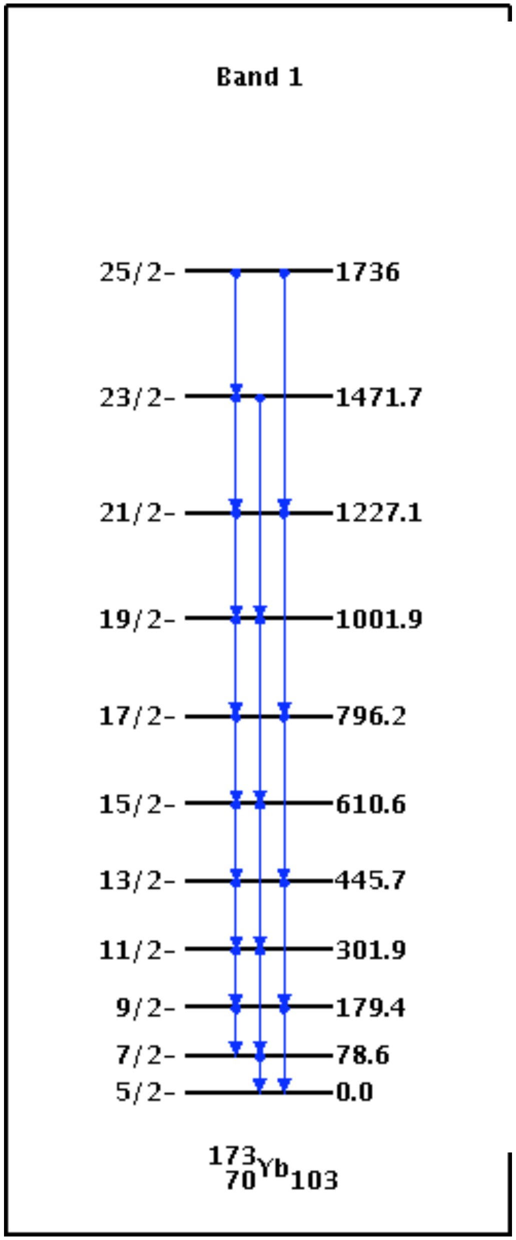
945.6

551.6

262.7

78.7

0.0



LO

1796.6

1515.9

1257.6

1021.8

808.5

617.6

449.1

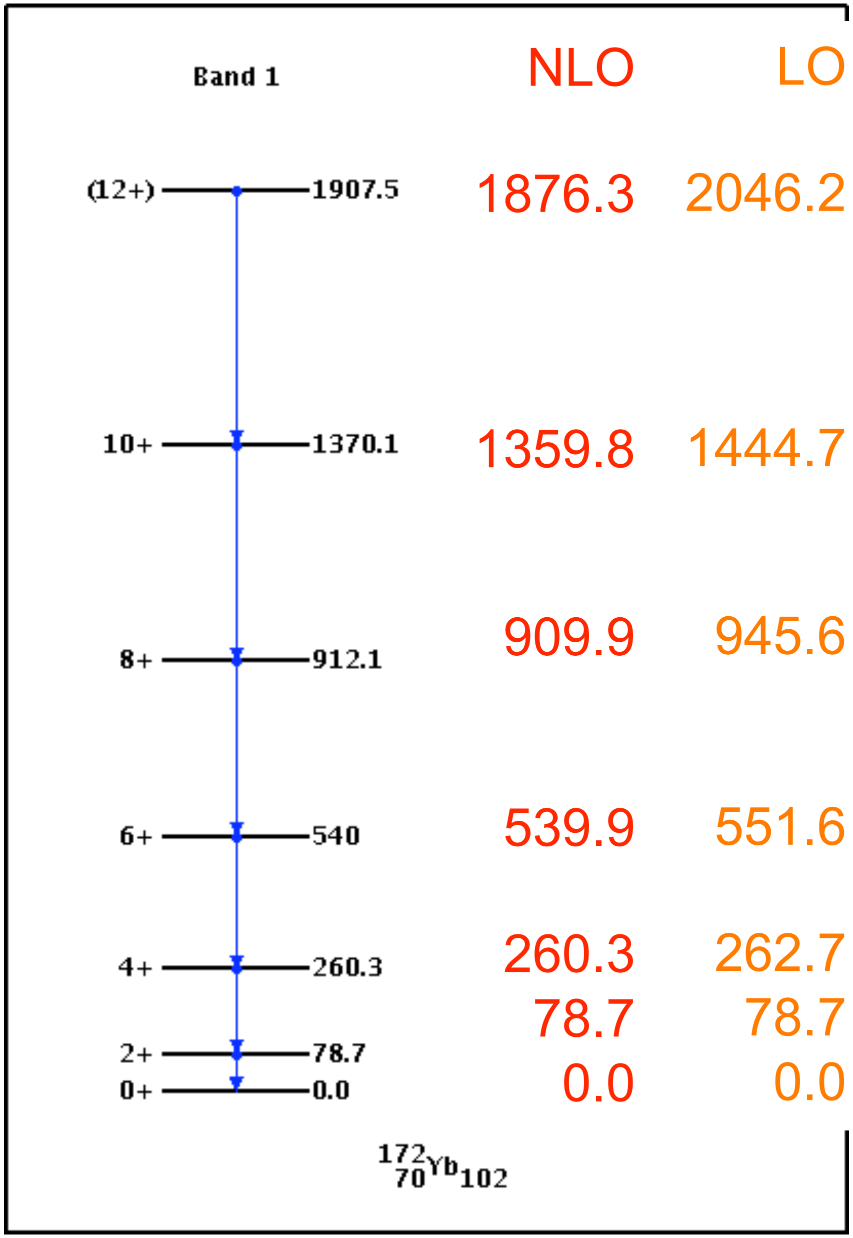
303.2

179.7

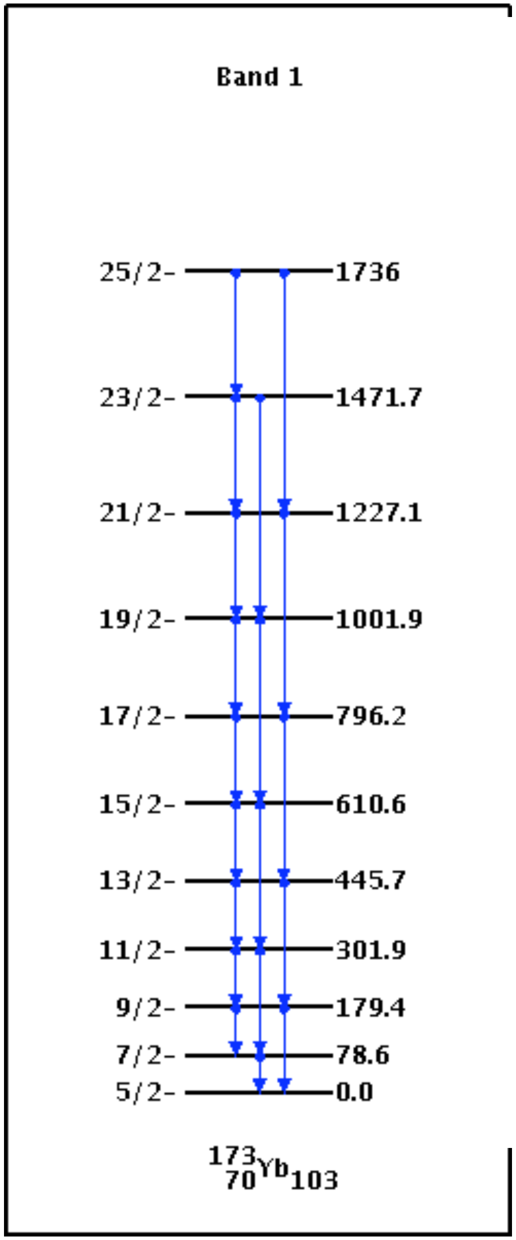
78.6

0.0

Comparison with data (energies in keV)



NLO	LO
1876.3	2046.2
1359.8	1444.7
909.9	945.6
539.9	551.6
260.3	262.7
78.7	78.7
0.0	0.0



NLO	LO
1752.9	1796.6
1485.0	1515.9
1236.6	1257.6
1008.2	1021.8
800.1	808.5
612.9	617.6
446.8	449.1
302.2	303.2
179.4	179.7
78.6	78.6
0.0	0.0

Breakdown for angular momenta $I \approx \Omega/\xi$

3. Construct the most general Hamiltonian consistent with the symmetry and the symmetry breaking

Nonlinear realization of (rotational) symmetry

Weinberg 1967; Coleman, Callan, Wess & Zumino 1969

Assume ground state is invariant only under rotations around the z-axis. Rotations of that state cannot be distinguished if they differ only by a rotation around the z-axis. (NG-modes parameterize the coset SO(3)/SO(2))

Rotation in terms of Euler angles

$$r(\alpha,\beta,\gamma) = e^{-i\alpha \hat{J}_z} e^{-i\beta \hat{J}_y} e^{-i\gamma \hat{J}_z}$$

Rotations that change the ground state: Nambu-Goldstone modes

$$g(\alpha,\beta) = e^{-i\alpha \hat{J}_z} e^{-i\beta \hat{J}_y}$$

Transformation properties under rotations

$$rg = \tilde{g}(g,r) h(g,r)$$

$$h = \exp\left(-i\varphi(g,r)\hat{J}_z\right)$$

Nonlinear realization of the rotational symmetry

1. E_x and E_y transform as the x and y-components of a vector under rotations.

$$\begin{aligned} E_x &= \dot{\alpha} \sin \beta \\ E_y &= -\dot{\beta} \end{aligned}$$

2. The “covariant derivative” D_t transforms as the z-component of a vector under rotations.

$$D_t \equiv \partial_t - i E_z J_z$$

$$E_z = -\dot{\alpha} \cos \beta$$

Any Lagrangian consisting of combinations of E_x, E_y, and D_t (acting on other fields) **that is formally invariant under SO(2)** (i.e. rotations around the z-axis) **is indeed invariant under SO(3)**.

Weinberg (1967); Coleman, Wess, Zumino (1969); Callan, Coleman, Wess & Zumino (1969). **Pedagogical reviews**: S. Weinberg, The Quantum Theory of Fields, Vol.II, chap. 19: C. P. Burgess, Physics Reports 330 (2000) 193; T. Brauner, arXiv:1001.5212.