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Gravity Effects on Nuclear Reactions at Low Energies*

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We consider the effect of the Earth gravitational attraction on nuclear reactions at very low energies. In particular, the Mott oscillation characteristic of scattering of two identical nuclei is found to be slightly but significantly modified. The effect discussed here is different but complementary to the COW effect in cold neutron interferometry. We take as an example $^{208}Pb+^{208}Pb$. An angle precision of about 10^{-3} deg. is required to detect the effect.

Several years ago we proposed a way of checking whether a long range color van der Waals (CVDW) force may be operating between hadrons [1]. The system $^{208}Pb+^{208}Pb$ at very low bombarding energies was chosen for the purpose. The effect of the above force is to slightly change the Mott oscillatory pattern.

An angle precision of about 10^{-3} deg. is required to see the effect. Such an experiment with this angle precision was performed at GANIL [2] and an a new upper bound for the strength of the CVDW force was established.

Lowering the bombarding energy would allow the establishment of a more stringent upper bound. On the other hand other effects such as the Earth attraction may become important. The purpose of this short note is to assess the importance of the gravitational attraction on nuclear reactions at low energies. Since the effect is more easily seen if it manifests itself as interference in the cross-section, we restrict our discussion to the scattering of identical nuclei. We remind the reader that gravitational effects have been considered before with regards to very cold neutrons [3,4].

In the pioneering experiment of R. Colella, A.W. Overhauser and S.A. Werner [3], (COW - effect for short) a beam of cold neutrons is split into two beams as it passes through a crystal. The resulting two "beams" of neutrons travel different routes and as they are detected, they exhibit interference effects owing to the phase difference arising from gravity being operative differently along the two routes.

The gravity effect we wish to discuss here is a bit different from the COW effect. The use of identical particles simulate, to an extent, the two neutron beams. The gravity effect on the center of mass, as we show below, is quite small. On the other hand, though not always appreciated, gravity influences indirectly the relative motion through the small effect it inflicts on the momentum through energy conservation. This *new* effect of gravity, should be observed in the low energy scattering of identical nuclei or, for that matters, other systems such as atoms or molecules.

From the theory point of view, the problem we are discussing here is, to be precise, a three-body problem which is simplified a lot owing to the disparity in masses. Thus we end up in solving the scattering problem of two particles in the presence of an external field. This problem occurs is several areas. In nuclear physics, we mention the case of Hambury-Brown Twiss interferometry involving the emission of two proton from a hot nucleus. The two protons, as they escape to the detector will feel the effect of the Coulomb repulsion coming from the hot source and a proper treatment of this external field is usually required.

The test of the theory presented here will be of value in so far as it will supply confidence in the treatment of the influence of gravity on Mott oscillations in a semi quantal framework. Further, the measurement of the effect will supply one more and *different* example of how gravity and quantum mechanics meet in a micro-macro world.

In the scattering of two identical nuclei we have to consider two processes, depicted in Fig. 1. The projectile is scattered and then detected. The second process involves the detection of the target after the scattering. If the scattering plane is taken to be perpendicular to the surface of the Earth, these two processes occur at different altitudes and accordingly are affected differently by the gravitational attraction. Further the center of mass of the system suffers acceleration. The quantum mechanical amplitude describing the scattering process under these circumstances may be written as:

$$f_{sy}(\theta) = e^{i\delta^{(\theta)}_{c.m.}} f(\theta) e^{i(k_\theta r - \eta \ln 2k_\theta r)} + e^{i\delta^{(\pi-\theta)}_{c.m}} f(\pi-\theta) e^{i(k_{(\pi-\theta)} r - \eta \ln 2k_{\pi-\theta} r)} \qquad (1)$$

where the phase $\delta^{(\theta)}_{c.m.}$ corresponds to the effect of gravity on the centre of mass of the two colliding nuclei and is given, using the WKB approximation, by

$$\delta^{(\theta)}_{c.m.} = \frac{2mg^2}{3\hbar} T^3_\theta \qquad (2)$$

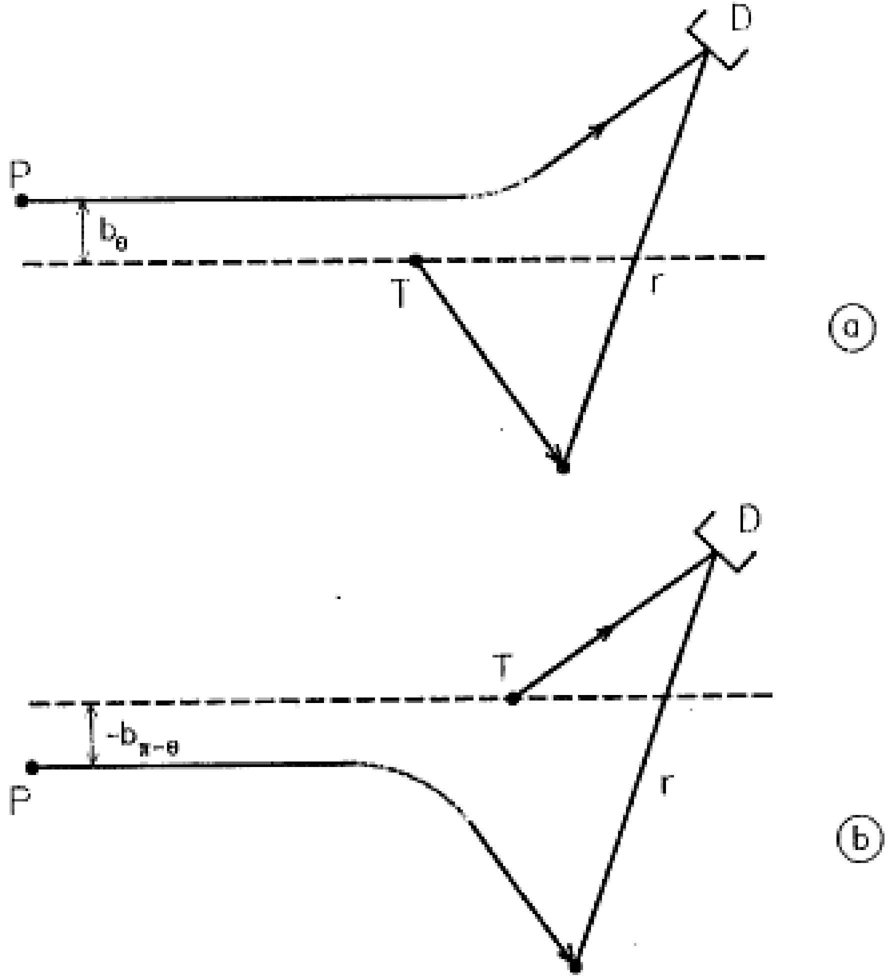


Figure 1. The two trajectories contributing to the scatttering of the identical nuclei.

The label $\theta(\pi-\theta)$ represents the fact that the time, $T_\theta(T_{\pi-\theta})$, it takes the projectile (target) to reach the detector depends on $\theta(\pi-\theta)$ through the impact parameter $b_{\theta(\pi-\theta)}$. The relative wave number k_θ is also different for the two trajectories considered and therefore, the exponentials associated with the scattered spherical waves must be included in $f_{sy}(\theta)$. In equation (2), m is the mass of ^{208}Pb and g is the gravitational acceleration ($g = 980$ cm s^{-2}).

The cross section may be written as, with $f = \sqrt{\frac{d\sigma_{cl}(\theta)}{d\Omega}}\, e^{i\psi(\theta)}$

$$\frac{d\sigma}{d\Omega} = |f_{sy}|^2 = \frac{d\sigma_{cl}(\theta)}{d\Omega} + \frac{d\sigma_{cl}(\pi-\theta)}{d\Omega}$$

$$+ 2\sqrt{\frac{d\sigma_{cl}(\theta)}{d\Omega}\frac{d\sigma_{cl}(\pi-\theta)}{d\Omega}} \cos[(\psi(\theta)-\psi(\pi-\theta))+(\delta^{(\theta)}_{c.m.} - \delta^{(\pi-\theta)}_{c.m.})$$

$$+ (k_\theta - k_{\pi-\theta})r - \eta \ln\left(\frac{k_\theta}{k_{\pi-\theta}}\right)\Big] \qquad (3)$$

where η is the Sommerfield parameter.

The times T_θ and $T_{\pi-\theta}$ can be evaluated classically from (see Fig. 2)

$$T_\theta = \frac{D}{V^{(\theta)}_{c.m.}} = \frac{L_0 + L \sec\frac{\theta}{2}}{v^{(\theta)}} = \frac{L_0 + L \sec\frac{\theta}{2}}{v\left(1 - \frac{2gb_\theta}{v^2}\right)^{1/2}}$$

$$T_{\pi-\theta} = \frac{D}{V^{(\pi-\theta)}_{c.m.}} = \frac{L_0 + L \operatorname{sen}\frac{\theta}{2}}{v^{(\pi-\theta)}} = \frac{L_0 + L \sec\frac{\theta}{2}}{v\left(1 - \frac{2gb_{\pi-\theta}}{v^2}\right)^{1/2}} \qquad (4)$$

$$k_\theta = \frac{mv_\theta}{\hbar} = \frac{mv}{\hbar}\left(1 - \frac{2gb_\theta}{v^2}\right)^{1/2}$$

$$k_{\pi-\theta} = \frac{mv_{\pi-\theta}}{\hbar} = \frac{mv}{\hbar}\left(1 + \frac{2gb_{\pi-\theta}}{v^2}\right)^{1/2} \quad (5)$$

$$\frac{r}{2} = \frac{\frac{L}{2}}{\cos\frac{\theta}{2}} \qquad b_\theta = \frac{\eta}{k}\cot\frac{\theta}{2} \quad (6)$$

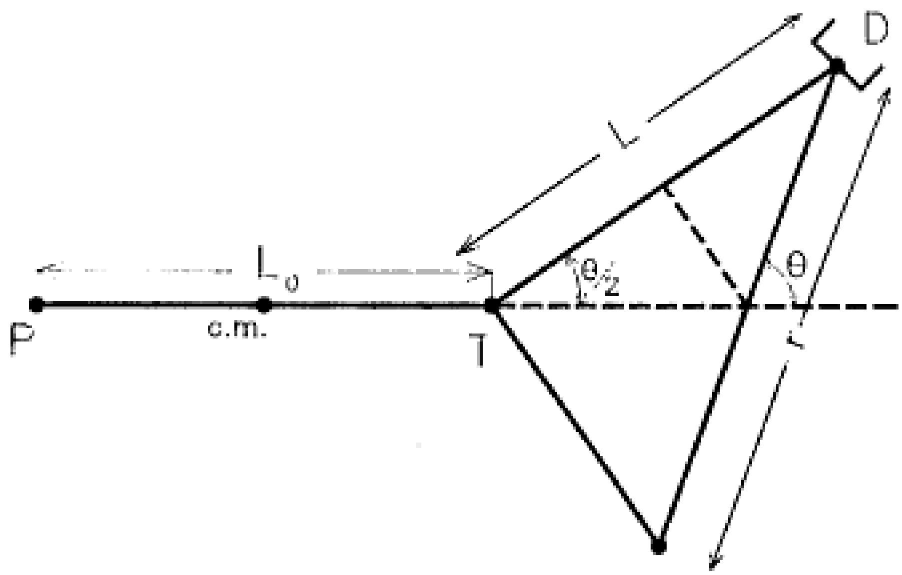


Figure 2. The geometrical variables used in calculating Eqs. (4), (5) and (6).

In above equations, L_o and L represent the distance from the injection point to the target and from the target to the detector, respectively.

Thus the phase in (3) arising from gravity can be written as

$$\Delta\delta^{gravity} = \delta(\theta)_{c.m.} - \delta(\pi-\theta)_{c.m.} + (k_\theta - k_{\pi-\theta})r - \eta \ln\frac{k_\theta}{k_{\pi-\theta}}$$

$$\cong \frac{2mg^3\left(L_0 + L\sec\frac{\theta}{2}\right)^3}{\hbar v^5}(b_\theta + b_{\pi-\theta})$$

$$+ \frac{mgL\sec\frac{\theta}{2}}{\hbar v}(b_\theta + b_{\pi-\theta}) + \frac{\eta g}{v^2}(b_\theta + b_{\pi-\theta})$$

$$= \frac{2mg^3\left(L_0 + L\sec\frac{\theta}{2}\right)^3}{\hbar v^5}\frac{d}{\sin\theta} + \frac{mgL\sec\frac{\theta}{2}}{\hbar v}\frac{d}{\sin\theta} + \frac{\eta g}{v^2}\frac{d}{\sin\theta}, \quad (7)$$

where we have used the Rutherford relations, $b_\theta = \frac{d}{2}\cot\frac{\theta}{2}$ and $b_{\pi-\theta} = \frac{d}{2}\tan\frac{\theta}{2}$. Here d is the distance of closest approach for head-on collision, $d = \frac{Z_1 Z_2 e^2}{E}$.

For small enough angles, $\sec\frac{\theta}{2} \approx 1$, and we can approximate (7) by

$$\Delta\delta^{gravity} = \frac{mgLd}{\hbar v}\frac{1}{\sin\theta}\left\{1 + \frac{\hbar\eta}{Lvm} + \frac{2g^2}{v^4}\frac{(L_0+L)^3}{L}\right\} \quad (8)$$

The phase difference $\psi(\theta)-\psi(\pi-\theta)$ of Eq. (3) for Coulomb scattering is given by

$$\psi(\theta)-\psi(\pi-\theta) = 2\eta \ln\cot\frac{\theta}{2} \quad (9)$$

Therefore the period of the Mott oscillation for large η can be written as

$$P^0_{Mott} = \frac{\pi}{\eta} \sin\theta \quad (10)$$

The gravity correction to this period goes as 1/sinθ (Eq. 8), and then the period becomes (taking the leading term in Eq. (8))

$$P^{(+)}_{Mott} = \frac{\pi}{\eta} \sin\theta \left[\frac{1}{1 - \frac{gL}{v^2} \cot\theta} \right] (11)$$

Eq. (11) shows that the effect of gravity on the period is most effective at small angles (or close to 180°).

A judicious choice of L, θ and v is needed to obtain large enough effect that can be measured. One possible set up that would double the effect is to have two detectors placed in a symmetrical way at altitudes equidistant from the beam direction, one above and one below. The period (11), refers to detection at the higher altitude detector. The lower altitude detector should "see" Mott oscillation with a period

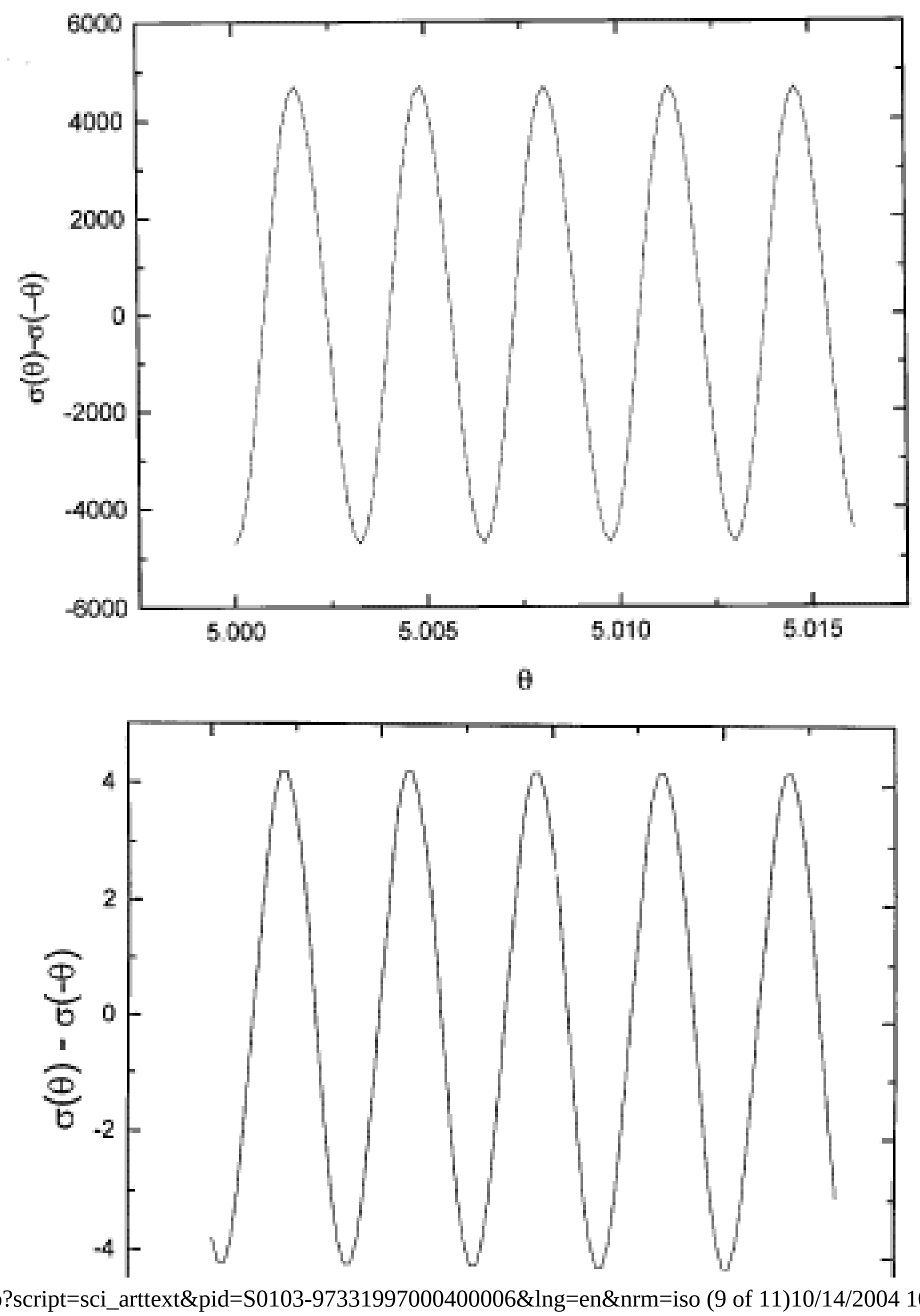
$$P^{(-)}_{Mott} = \frac{\pi}{\eta} \sin\theta \left[\frac{1}{1 + \frac{gL}{v^2} \cot\theta} \right] (12)$$

Thus, the difference in the periods $P^{(+)}$ and $P^{(-)}$, to leading order in g, is given by

$$\Delta P_{gravity} = \left(\frac{\pi g L \hbar \sqrt{M}}{Z_1 Z_2 e^2 \sqrt{E}} \right) \cos\theta , (13)$$

where M is the mass of the nucleus. Equation (13) shows a nice "encounter" of gravity (g), electromagnetism (e^2) and quantum mechanics ($\hbar$). A large $\Delta P_{(gravity)}$ can be obtained if $\sqrt{\frac{M}{E}} \frac{1}{Z_1 Z_2} \cos\theta$ were made as large as possible. One case could be very low energy elastic scattering of small-charge heavy identical molecules in the forward or backward direction. Not so heavy nuclei can also be a candidate.

For the purpose of illustration, we show in Fig. 3 the difference in the cross-section, $\sigma(\theta)$-$\sigma(-\theta)$ for the system ^{208}Pb+^{208}Pb at E_{CM} = 5 MeV (E_{lab} = 10 MeV) in two angular regions and $\theta_{CM} \approx 5^0$(θ_{Lab} = 2.5^0) and $\theta_{CM} \approx 82^0$(θ_{Lab} = 41^0). The length L was taken to be 10 meters. Thus one may detect the oscillation due to gravity by having a detector at θ_{Lab} = 41^0 and another one at θ_{Lab} = -41^0 in the vertical plane. The period here is about 0.005^0. Notwithstanding the influence of other charges that might be present in the neighbourhood, as discussed in details in Ref.[6], in the cross-section, the difference $\sigma(\theta)$-$\sigma(-\theta)$ should, to lowest order, remove these effects. It is clear from the figure that, though small, the effect may be as measurable as e.g. parity-non conservation in the nucleon-nucleon system [7].



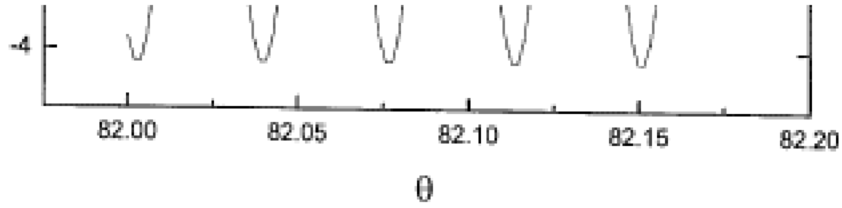


Figure 3. The cross-section difference for the system $^{208}Pb+^{208}Pb$ at E_{CM} = 5 MeV and E_{CM} = 80 MeV. See text for details.

In conclusion, though difficult, a measurement of the effect above would be extremely interesting and complementary to the COW effect. We believe this measurement is feasible with the angle measurement precision already attained at GANIL [2,5].

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